

B. Sc. DEGREE EXAMINATION, NOVEMBER 2016  
BRANCH I - MATHEMATICS  
FIRST SEMESTER

COURSE : MAJOR – CORE  
PAPER : DIFFERENTIAL CALCULUS  
TIME : 3 HOURS

MAX. MARKS : 100

SECTION – A (10X2=20)  
ANSWER ALL THE QUESTIONS

1. Write down the nth derivative of  $\sin(ax + b)$ .
2. If  $xy = ae^x + be^{-x}$ , prove that  $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - xy = 0$ .
3. Define envelope of the family of curves.
4. Find the envelope of the family of straight lines  $y = mx + \frac{a}{m}$ , the parameter being 'm'.
5. Write down the formula for the coordinates of the centre of curvature of a curve.
6. Find the radius of curvature for the curve  $y = e^x$ .
7. Define evolute of a curve.
8. Define the chord of curvature.
9. Write down the condition for maxima and minima functions of two variables.
10. When do you say that a curve is symmetrical with respect to the  $x - axis$ ?

SECTION – B (5X8=40)  
ANSWER ANY FIVE QUESTIONS

11. Find  $y_n$ , if  $y = \frac{x^2}{(x+2)x-1}$ .
12. If  $x = \sin \theta$ ,  $y = \cos p\theta$ , prove that  $(1 - x^2) y_2 - xy_1 + p^2y = 0$ .
13. Find the envelope of the circles drawn on the radius vectors of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  as diameter.
14. Find the radius of curvature at any point of the cycloid  $x = a(\theta + \sin \theta)$  and  $y = a(1 - \cos \theta)$ .
15. Find the radius of curvature of the curve  $y^2 = \frac{a^2(a-x)}{x}$  at  $(a, 0)$ .
16. Find the radius of curvature of the curve  $r^n = a^n \cos n\theta$ .
17. Trace the curve  $y^2 = x^2 \frac{a+x}{b-x}$ .

**SECTION – C**  
**ANSWER ANY TWO QUESTIONS**

(2X20=40)

18. (a) If  $y = \sin(m \sin^{-1} x)$ , prove that

$$1 - x^2 y_{n+2} - 2n + 1 xy_{n+1} + m^2 - n^2 y_n = 0.$$

(b) Find the envelope of the straight lines  $\frac{x}{a} + \frac{y}{b} = 1$ , where the parameters are related by the equation  $a^2 + b^2 = c^2$  where  $c$  is a constant. (10 + 10)

19. (a) Find the coordinates of the centre of curvature of the curve  $y^2 = 4ax$  at the point 't'.

(b) Find the  $p - r$  equation of the curve  $r = a(1 - \cos \theta)$ . Hence deduce the radius of curvature. (10 + 10)

20. Discuss the maxima and minima of the function  $x^3 y^2 (6 - x - y)$ .

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