

B. Sc. DEGREE EXAMINATION, NOVEMBER 2016
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR – CORE
PAPER : ALGEBRA AND TRIGONOMETRY
TIME : 3 HOURS
MAX. MARKS : 100

SECTION – A (10X2=20)
ANSWER ALL THE QUESTIONS

1. Find the equation whose roots are the roots of $x^5 + 6x^4 + 6x^3 - 7x^2 + 2x - 1 = 0$ with the signs changed
2. Remove the fractional coefficients from the equation $x^3 - \frac{1}{4}x^2 + \frac{1}{3}x - 1 = 0$
3. Diminish by 3 the roots of the equation $x^5 - 4x^4 + 3x^2 - 4x + 6 = 0$.
4. Find the nature of the roots of the equation $x^6 + 3x^2 - 5x + 1 = 0$
5. Define orthogonal matrices
6. State Cayley Hamilton Theorem
7. Express $\cos^5 \theta$ in series of cosines of multiples of θ
8. Prove that $\cosh 2x = \cos h^2 x + \sin h^2 x$
9. If $\sin A + iB = x + iy$, prove that $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$
10. Show that $\log 1 + i \tan \alpha = \log \sec \alpha + i\alpha$

SECTION – B (5X8=40)
ANSWER ANY FIVE QUESTIONS

11. Show that the roots of the equation $x^3 + px^2 + qx + r = 0$ are in arithmetic progression if $2p^3 - 9pq + 27r = 0$.
12. If α, β, γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$, form the equation whose roots are $\alpha\beta, \beta\gamma$ and $\gamma\alpha$.
13. Calculate A^4 using Cayley Hamilton theorem given $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$
14. Find the Eigen values of the matrix $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$
15. If $\frac{\sin \theta}{\theta} = \frac{5045}{5046}$ show that $\theta = 1^\circ 58'$ approximately, given $\frac{\sin \theta}{\theta} \rightarrow 1$ as $\theta \rightarrow 0$, θ is measured in radians.
16. Separate into real and imaginary parts $\tanh(1 + i)$.
17. Express $\tanh^{-1} x$ in logarithmic form and deduce that

$$\tanh^{-1} x = x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \dots$$

SECTION – C
ANSWER ANY TWO QUESTIONS

(2X20=40)

18. a) Find the positive root of the equation $x^3 - 2x^2 - 3x - 4 = 0$ correct to three places of decimals using Horner's method.

b) Separate $\sin A + iB$ into real and imaginary parts. Hence show that if its modulus be unity, $\sinh^2 B - \cos^2 A = 0$. (15+5)

19. a) Show that the equation $x^4 - 3x^3 + 4x^2 - 2x + 1 = 0$ can be transformed into a reciprocal equation by diminishing the roots by unity. Hence solve the equation.

b) Find the value of $\text{Log} \frac{1 + \cos \theta + i \sin \theta}{\cos \theta - 1 + i \sin \theta}$. (10+10)

20. a) Diagonalize the matrix $A = \begin{matrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{matrix}$

b) If $\cosh u = \sec \theta$, show that $u = \log \tan \frac{\pi}{4} + \frac{\theta}{2}$ (12+8)

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