**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**
**(For candidates admitted during the academic year 2011 – 12 & thereafter)**

**SUBJECT CODE : 11MT/MC/RA54**

**B. Sc. DEGREE EXAMINATION, NOVEMBER 2016**
**BRANCH I - MATHEMATICS**
**FIFTH SEMESTER**

**COURSE : MAJOR – CORE**
**PAPER : REAL ANALYSIS**
**TIME : 3 HOURS** **MAX. MARKS : 100**

**SECTION – A** **(10X2=20)**
**ANSWER ALL THE QUESTIONS**

1. When is a function said to be non-decreasing and non-increasing and monotonic?
2. Define the continuity of a function $'f'$ at a point $a \in R^1$
3. When is a subset $A$ of $M$ said to be dense?
4. Define a limit point of a subset $E$ of a metric space $M$.
5. Define the Cauchy sequence in a metric space.
6. Define the continuity of $'f'$ on a metric spaces.
7. When is a metric space $S$ called connected?
8. Define the terms ‘fixed point’ and ‘contraction’.
9. State the second fundamental theorem of calculus.
10. Define ‘Riemann integral’.

**SECTION – B** **(5X8=40)**
**ANSWER ANY FIVE QUESTIONS**

11. If $f$ and $g$ are real valued functions, if $f$ is continuous at $'a'$ and $g$ continuous at $f\ a$ then show that $g$ is continuous at $'a'$.
12. Prove that every subset of $R_d$ is open.
13. Show that in a metric space $(X, d)$ a sequence $\{S_n\}$ converges to $'p'$ if, and only if, every subsequence of $\{S_n\}$ converges to $'p'$.
14. Prove that in any metric space $(X, d)$ every compact subset of $X$ is complete.
15. Let $f: S \to M$ be a function from a metric space $S$ to another metric space $M$. Let $X$ be a connected subset of $S$. If $f$ is continuous on $X$, then show that $f(X)$ is a connected subset of $M$.
16. State and prove the Rolle’s theorem.
17. Prove the additive property of the Riemann integral (ie) if $a < c < b$, then $_a^c f + \ _c^b f = \ _a^b f$.

**SECTION – C** **(2X20=40)**

**ANSWER ANY TWO QUESTIONS**

18. Let $< M, \rho >$ be the metric space and let $'a'$ be a point in $M$. Let $f$ and $g$ be real valued functions whose domains are subsets of $M$. If $\lim_{x\to a} f(x) = L$ abd $\lim_{x\to a} g(x) = M$ then

   a) $\lim_{x\to a} (f(x) + g(x)) = L + M$

   b) $\lim_{x\to a} (f(x) - g(x)) = L - M$

   c) $\lim_{x\to a} (f(x)g(x)) = LM$

   d) $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L}{M} (M \neq 0)$.

19. a) State and prove the mean value theorem. (10)

   b) If $G_1$ and $G_2$ are the open subsets of a metric space $M$ then prove that $G_1 \cap G_2$ is also open in $M$. (10)

20. a) Prove that "In an Euclidean space $R^k$ , every Cauchy sequence is convergent". (10)

   b) State and prove the Fixed-point theorem. (10)

⅄⅄⅄⅄⅄⅄⅄⅄⅄