STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2011–12 & thereafter)

SUBJECT CODE : 11MT/MC/AS54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2016
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : ALGEBRAIC STRUCTURES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

Answer all questions: **(10 X 2 =20)**

1. Let $G=\{1, \frac{-1}{2}+i\frac{\sqrt{3}}{2}, \frac{-1}{2}-i\frac{\sqrt{3}}{2}\}$. Is G a group under usual multiplication of complex numbers?
2. Show that if every element of a group G is its own inverse, then G is abelian.
3. Define a normal subgroup of a group.
4. If $\varphi : G \to G'$ is a homomorphism from a group G into a group G' then prove that $\varphi(e) = e'$, where e and e' are respectively the identity elements of G and G'.
5. Write the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 8 & 1 & 6 & 4 & 7 & 5 & 9 \end{pmatrix}$ as a product of disjoint cycles.
6. Define an automorphism of a group G.
7. Give an example of a ring which is not an integral domain.
8. Prove that every field is an integral domain.
9. Define an ideal of a ring.
10. What is the field of quotients of the ring of integers?

SECTION – B

Answer any five questions: **(5 X 8 = 40)**

11. If G is a group of even order, prove that it has an element $a \neq e$ satisfying $a^2 = e$.
12. Prove that a subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.
13. If G is a group, prove that $A(G)$, the set of automorphisms of G, is also a group.
14. Prove that a finite integral domain is a field.
15. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.
16. Let R be a commutative ring with unit element and M is an ideal of R. Prove that M is a maximal ideal of R if and only R/M is a field.
17. If G is a group and if H is a subgroup of H of index 2 in G, prove that H is a normal subgroup of G.

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SECTION – C

Answer any two questions: **(2 x 20 = 40)**

18. (a) Let G be the group of integers under addition and $H=\{0,\pm 5,\pm 10,\pm 15,\ldots\}$ be the set of multiples of 5. Is H a subgroup of G ? If so write down all the cosets of H in G.

(b) Let $\phi:G\to\overline{G}$ be a homomorphism of G onto $\overline{G}$ with kernel K. The prove the following.

(i) K is a normal subgroup of G.

(ii) $G/K \cong \overline{G}$. (4+10)

19. (a) State and prove Cayley's theorem.

(b) If $\varphi: R \to R'$ is a homomorphism of ring, prove that the kernel of φ, $I(\varphi)$, is an ideal of R.

(c) Give an example of division ring which is not a field. (12+5+3)

20. (a) Prove that any integral domain can be imbedded in a field.

(b) Give an example of a prime ideal of the ring J of integers which is not a maximal ideal of J.

(c) Give an example of an infinite cyclic group. (15 +3+2)