STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015-16)

SUBJECT CODE : 15MT/MC/ML24

B. Sc. DEGREE EXAMINATION, APRIL 2015
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : MAJOR CORE
PAPER : MULTIPLE INTEGRALS AND LAPLACE TRANSFORMS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION A

Answer All Questions: **10 x 2 = 20**

1. Evaluate $\int_0^3 \int_1^2 x\,y\ \ x+y\ \ dy\,dx$
2. Evaluate $\int_{-\pi}^{\frac{\pi}{2}} \int_0^{2\cos\theta} r^2\ dr\,d\theta$
3. Change the following integral into polar coordinates
$$\int_0^\infty \int_0^\infty e^{-\ x^2\ +\ y^2}\ dx\,dy$$
4. Find the Jacobian of x, y with respect to r,θ if $x = r\ cos\ \theta$, $y = r\ sin\ \theta$
5. Prove that $\beta\ \ m, n\ = \beta\ \ n, m$
6. Find the value of $\Gamma(1)$.
7. Find L [e^{-at}]
8. Find L [$\sin^2 2t$]
9. Find L^{-1} [$\frac{1}{(s-3)^5}$]
10. Find L^{-1} [$\frac{s}{(s+2)^2}$]

SECTION B

Answer Any Five Questions: **5 x 8 = 40**

11. Evaluate $x\,y\,dx\,dy$ taken over the positive quadrant of the circle $x^2 + y^2 = a^2$.
12. By changing into polar coordinates evaluate the integral
$$\int_0^{2a} \int_0^{\sqrt{2\,a\,x - x^2}} x^2\ +\ y^2\ \ dx\,dy$$
13. Evaluate $x\,y\,z\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

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14. Evaluate $\int_0^1 x^m \left(\log\frac{1}{x}\right)^n dx$.

15. Find $L\,[\,t\,e^{-t}\sin t\,]$.

16. Find $L^{-1}\left\{\frac{1}{(s+1)(s^2+2s+2)}\right\}$.

17. Find $L^{-1}\left\{\frac{s+2}{(s^2+4s+5)^2}\right\}$

SECTION C

Answer Any Two Questions: **2 x 20= 40**

18. (a) Change the order of integration in the integral

$\int_0^a \int_{\frac{x^2}{a}}^{2a-x} x\,y\,dx\,dy$ and evaluate it.

(b) Evaluate $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ taken over the volume bounded by the planes $x = 0, y = 0, z = 0, x + y + z = 1.$

19. (a) Find the relation between Beta and Gamma function

(b) Express $\int_0^1 x^m (1-x^n)^p\,dx$ in terms of Gamma function and evaluate the integral $\int_0^1 x^5 (1-x^3)^{10}\,dx$.

20. (a) If $L\,[f(t)] = F(s)$ and if $\frac{f(t)}{t}$ has a limit as t → 0 , then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)ds$ and hence find $L\left\{\frac{\sin at}{t}\right\}$.

(b) Solve the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$ given that $y = \frac{dy}{dt} = 0$ when $t = 0$.