STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/MC/VL64

B. Sc. DEGREE EXAMINATION, APRIL 2016
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : VECTOR SPACES AND LINEAR TRANSFORMATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS. **(10X2=20)**

1. Define a vector space over a field F.
2. Give an example of a subspace of the vector space $F[x]$, of all polynomials in x over the field F.
3. Define the dual space of a vector space V.
4. Give any two bases of the space R^2 over the field R of real numbers.
5. Define the orthogonal complement $W^{\perp}$ of a subspace W of an inner product space V.
6. Give an orthonormal basis of the inner product space R^2 over R.
7. Define a singular linear transformation.
8. If T is identity transformation on a vector space V of dimension n, then what is the rank of T ?
9. When do you say a matrix is orthogonally diagonalizable?
10. Define similar matrices.

SECTION –B

ANSWER ANY FIVE QUESTIONS. **(5X8=40)**

11. Let V be a vector space over a field F and **0** be the zero element of V. Prove that
    (i) $\alpha 0 = 0$, for every $\alpha \in F$
    (ii) $ov = 0$, for every v in V .
    (iii) $\alpha \; -v \;\; = -(\alpha v)$, for every $\alpha \in F, v \in V$.
    (iv) If $v \neq 0$, then $\alpha v = 0$ implies that $\alpha = 0$.
12. If $v_1, v_2, \cdots, v_n$ is a basis of V over F and if independent $w_1, w_2, \cdots, w_m$ in V are linearly independent over F, then prove that $m \leq n$.
13. Find an orthonormal basis for the inner product space of polynomials in x of degree 2 or less over the field R of real numbers.
14. Prove that the characteristic vectors corresponding to distinct characteristic roots of $T \in A(V)$ are linearly independent in V.

15. Show that the matrix $A = \begin{pmatrix} 5 & -3 \\ 3 & -1 \end{pmatrix}$ is not diagonalizable.
16. State and prove Schwartz inequality.
17. Prove that any orthonormal set of vectors in an inner product space are linearly independent.

SECTION –C

ANSWER ANY TWO QUESTIONS. **(2X20=40)**

18. (a) If $v_1, v_2, ..., v_n \in V$ are linearly independent, then prove that every element in their linear span has a unique representation in the form $\lambda_1 v_1 + ... + \lambda_n v_n$.

    (b) If $V = R^3$ is 3-dimensional Euclidean space and if $S =$ 1,0,0 , 0,1,0 , 0,0,1 . Find $L(S)$.

    (c) If V is a finite dimensional space over a field F and W is a subspace of V, then prove that W is also finite dimensional, $\dim(W) \leq \dim(V)$ and $\dim(V/W) = \dim(V) - \dim(W)$.

    (5+5+10)

19. (a) Prove that any finite dimensional inner product space V has an orthonormal basis.

    (b) If V is finite dimensional over F and if $S, T \in A(V)$, then prove that

    (i) $r(ST) \leq r(T)$

    (ii) $r(TS) \leq r(T)$

    (iii) $r(ST) = r(TS) = r(T)$ for S regular in $A(V)$

    (iv) If $T \in A(V)$ and if $S \in A(V)$ is regular, then prove that $r(T) = r(STS^{-1})$. (10+10)

20. (a) Orthogonally diagonalize the symmetric matrix $A = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$.

    (b) Is union of two subspaces of a vector space V a subspace of V ? If the answer is yes, prove your answer. If the answer is no, then state and prove the necessary and sufficient condition(s) for union two subspaces of V to be a subspace of V. (10+10)