STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/MC/SF44

B. Sc. DEGREE EXAMINATION, APRIL 2016
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : MAJOR CORE
PAPER : SEQUENCES AND SERIES, FOURIER SERIES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(10×2=20)**

1. Define countable set. Give an example.
2. Define upper bound.
3. Define convergent sequence.
4. Define a bounded sequence. Give an example of a bounded sequence.
5. Define limit of a sequence.
6. Define Cauchy sequence.
7. Prove that $\sum_{n=1}^{\infty}(-1)^n$ diverges.
8. Define conditionally convergence of a series.
9. Define odd function.
10. Define Fourier series.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5×8=40)**

11. Prove that the inverse image of the union of two sets is the union of the inverse image.
12. If $\{S_n\}_{n=1}^{\infty}$ is a sequence of nonnegative numbers and if $\lim_{n\to\infty} S_n = L$ then prove L ≥ 0.
13. Prove that the series $\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\right)$ is converges.
14. If $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers, then prove that $\{S_n\}_{n=1}^{\infty}$ is bounded.
15. Prove that the series $\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)$ is divergent.
16. Prove that a non-decreasing sequence which is not bounded above diverges to infinity.
17. Express $f(x)=\frac{1}{2}(\pi - x)$ as a Fourier series with period 2π to be valid in $(0, 2\pi)$.

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2×20=40)**

18. a) If A_1, A_2 ,.... are countable sets then prove that $\bigcup_{n=1}^{\infty} A_n$ is countable.

    b) Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent.

    c) If $\lim_{n\to\infty} S_n = L$ and $\lim_{n\to\infty} S_n = M$ then prove that L = M.

19. a) If $\{a_n\}_{n=1}^{\infty}$ is a non-increasing sequence of positive numbers and if $\sum_{n=1}^{\infty} 2^n a_{2^n}$ converges then prove that $\sum_{n=1}^{\infty} a_n$ converges.

    b) State and prove the root test for absolute convergence of an infinite series.

20. a) Find the Fourier series for $f(x) = \begin{cases} 1+x & 0 < x < \pi \\ -1+x & -\pi < x < 0 \end{cases}$

    b) Find cosine series for $f(x) = \begin{cases} x & 0 < x < \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} < x < \pi \end{cases}$.

⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄