

B. Sc. DEGREE EXAMINATION, APRIL 2016
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : MAJOR CORE
PAPER : LAPLACE TRANSFORMS & PARTIAL DIFFERENTIAL EQUATIONS
TIME : 3 HOURS MAX. MARKS : 100

SECTION – A

ANSWER ALL THE QUESTIONS: (10×2=20)

1. Find $L(\cosh at)$.
2. Find $L(t^2 + 2t + 3)$.
3. Find $L^{-1} \frac{s-3}{s-3^2+4}$.
4. Find $L^{-1} \frac{s}{s+2^2}$.
5. Form a partial differential equation by eliminating arbitrary constants from $z = (x^2 + a)(y^2 + b)$.
6. Form a partial differential equation by eliminating arbitrary functions from $z = f(x + iy) + F(x - iy)$.
7. Find the complete integral of $p^2 + q^2 = x + y$.
8. Find the complete integral of $z = px + qy + c \sqrt{1 + p^2 + q^2}$.
9. Solve $r = a^2 t$.
10. Solve $(25D^2 - 40DD' + 16D'^2)z = 0$.

SECTION – B

ANSWER ANY FIVE QUESTIONS: (5×8=40)

11. What is the Laplace transform of the function $f(t) = t(0 < t < b)$
 $= 2b - t(b < t < 2b)$
12. Find (a) $L \frac{\sin at}{t}$ (b) $\int_0^{\infty} \frac{e^{-t} - e^{-2t}}{t} dt$
13. Find (a) $L^{-1} \frac{s-3}{s^2+4s+13}$ (b) $L^{-1} \frac{1}{s^2+a^2}$
14. Solve $(y + z)p + (z + x)q = x + y$.
15. Find the complete solution of $x^2 p^2 + y^2 q^2 = z^2$.
16. Solve $9(p^2 z + q^2) = 4$.
17. Solve $(D^2 - 2DD' + D'^2)z = 12xy$.

SECTION – C

ANSWER ANY TWO QUESTIONS:

(2×20=40)

18. Solve the simultaneous equations

$$\frac{dx}{dt} - \frac{dy}{dt} - 2x + 2y = 1 - 2t.$$

$$\frac{d^2x}{dt^2} + 2 \frac{dy}{dt} + x = 0 \text{ given } x = 0 = y, \frac{dx}{dt} = 0 \text{ when } t = 0.$$

19. (a) Solve the differential equation $\frac{d^2y}{dt^2} + 2 \frac{dy}{dt} - 3y = \sin t$ given that $y = 0, \frac{dy}{dt} = 0$ when $t = 0$.

(b) Solve $(x^2 - yz) p + (y^2 - zx) q = z^2 - xy$. (10 + 10)

20. (a) Solve $z^2 p^2 + q^2 = x^2 + y^2$.

(b) Solve $(2D^2 - 5DD' + 2D'^2)z = 24(y - x)$ (10 + 10)

▲▲▲▲▲▲▲▲▲▲▲▲