STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011–12 & thereafter)

SUBJECT CODE : 11MT/MC/GC64

B. Sc. DEGREE EXAMINATION, APRIL 2016
BRANCH I – MATHEMATICS
SIXTH SEMESTER

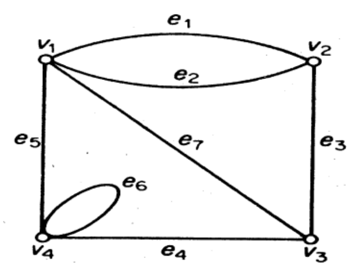
COURSE : MAJOR CORE
PAPER : GRAPH THEORY AND COMBINETORICS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A

ANSWER ALL QUESTIONS: **10 X 2 = 20**

1. Define automorphism of a graph.

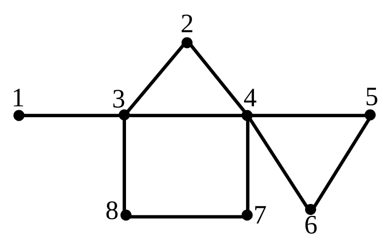
2. Find the incidence matrix for the following graph.



3. Define cut point and bridge of a graph.

4. Draw all trees with six vertices.

5. Define chromatic number of a graph. Also find the chromatic number of the following graph.



6. State five - colour theorem.

7. State the pigeonhole principle.

8. Find the number of 7-digit and 8-digit palindromes, under the restriction that no digit may appear more than twice.

9. Find the sequence corresponding to the ordinary generating function $2x^2(1-x)^{-1}$.

10. Find the exponential generating function of $0, 1, 2a, 3a^2, 4a^3, \ . \ . \ . \quad .$

SECTION-B

ANSWER ANY FIVE QUESTIONS: 5 X 8 = 40

11. Prove that any self complementary graph has $4n$ or $4n+1$ points.
12. Explain any four binary operations on two graphs G_1 and G_2 with an illustration.
13. Prove that a closed walk of odd length contains a cycle.
14. (a) Define centre of a tree.
    (b) Prove that every tree has a centre consisting of either one point or two adjacent points.
15. Prove: $1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$.
16. Find the number of solutions in integers of the equation $a+b+c+d=17$, where $1\le a\le 3, 2\le b\le 4, 3\le c\le 5, 4\le d\le 6$.
17. Tabulate D_n and T_n for n = 1(1)10.

SECTION-C

ANSWER ANY TWO QUESTIONS: 2 X 20 = 40

18. (a) Prove that a graph *G* with at least two points is bipartite if and only if all its cycles are of even length.
    (b) If *G* is a graph with $p\ge 3$ vertices and $\delta\ge p/2$, Prove that *G* is Hamiltonian.
    (10+10)

19. (a) Let *G* be a (*p*, *q*) graph. Prove that the following statements are equivalent.
    (i) *G* is a tree.
    (ii) Every two points of *G* are joined by a unique path.
    (iii) *G* is connected and *p* = *q* + 1.
    (iv) *G* is acyclic and *p* = *q* + 1.

    (b) State and prove Euler's formula for connected plane graph. (12+8)

20. (a) A chest contains 20 shirts, of which 4 are yellow, 7 are white and 9 are blue. At the least, how many shirts must one remove (blindfolded) to get $r=4,5,6,7,8,9$ shirts of the same colour?

    (b) Define Fibonacci sequence. Using its recurrence relation, show that

    $f(n)=\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1}-\left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]$, for *n* = 0, 1, 2,… (10+10)