STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011–12 & thereafter)

SUBJECT CODE : 11MT/MC/CA64

B. Sc. DEGREE EXAMINATION, APRIL 2016
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : COMPLEX ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A

ANSWER ALL QUESTIONS: **10 X 2 = 20**

1. State C – R equations in cartesion form and verify the same for $f(z) = z^3$.
2. Find the constant 'a' so that $u \;\; x, y \;\;\; = a\,x^2 - y^2 + \; xy$ is harmonic.
3. Define conformal and isogonal transformations.
4. Find the invariant points of $\mathrm{w} = \dfrac{1+\mathrm{z}}{1-\mathrm{z}}$.
5. State Cauchy's theorem.
6. Evaluate $\displaystyle\int_C \frac{\mathrm{z}\,\mathrm{dz}}{(9 - \mathrm{z}^2)(z+i)}$ where C is the circle $\;z\;\; = 2$ taken in the positive sense.
7. State Laruent's theorem.
8. Define zeroes of an analytic function and also find all zeroes of the function $\dfrac{z^3+1}{z^3-1}$.
9. Calculate the residue of $\dfrac{\mathrm{z}+1}{\mathrm{z}^2 - 2\mathrm{z}}$.
10. State Rouche's theorem.

SECTION-B

ANSWER ANY FIVE QUESTIONS: **5 X 8 = 40**

11. Show that $u = \log\sqrt{\mathrm{x}^2 + y^2}$ is harmonic and determine its harmonic conjugate, also determine the corresponding analytic function $f(z)$.
12. Show that the transformation $\mathrm{w} = \dfrac{5-4\mathrm{z}}{4\mathrm{z}-2}$ maps the unit circle $\;z\;\; = 1$ into a circle of radius unity and centre at $-½$.

13. Discuss the mapping $w = e^z$.

14. (i) State and prove Cauchy's inequality.

(ii) State and prove Liouville's theorem.

15. Evaluate $\int_C \frac{e^{2z}\,dz}{(z-1)^4}$ where C is $|z| = \frac{3}{2}$.

16. State and prove Argument theorem.

17. Evaluate: $\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta}$.

SECTION-C

ANSWER ANY TWO QUESTIONS: 2 X20 = 40

18. (a) State and prove the necessary condition for differentiability of a complex function.

(b) If $f(z) = u + iv$ is an analytic function and $u(x, y) = \frac{\sin 2x}{\cosh 2y + \cos 2x}$, find $f(z)$.

(c) Find the bilinear transformation which maps the points $z_1 = 0, z_2 = -i$, and $z_3 = -1$ into $w_1 = -1, w_2 = 1$ and $w_3 = 0$ respectively.

19. (a) State and prove Cauchy's Integral Formula.

(b) Find the Laruent's series expansion of the function $\frac{z^2-1}{(z+2)(z+3)}$ valid in the annular region $2 < |z| < 3$.

20. (a) State and prove Cauchy's residue theorem.

(b) Evaluate $\int_C \frac{dz}{z^3\,(z+4)}$ where C is $|z| = 2$.

(c) Evaluate: $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9}\,dx$.