STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-86.

MPhil DEGREE EXAMINATION-APRIL 2016

(Effective from the academic year 2015-2016)

Subject Code: 15MT/RO/FA205

Title: FUNCTIONAL ANALYSIS **Time: 3Hrs**

Core: OPTIONAL **Mark: 100**

SECTION -A

Answer any five questions **5×8=40**

1. Prove the completion of metric spaces l_p, $L_p(0,1)$ and $C[0,1]$
2. Define Haar measure. If X is a commutative Banach algebra, with an identity e, then prove that $\| e - x \| < 1$ implies x has an inverse.
3. Prove that the set of points in X for which the inverse exists is open.
4. Define invertible spectrum. Prove that $T: X \to X$ be a compact linear operator on a normed space X then for every $\lambda \neq 0$ the null space is finite dimensional.
5. Show that if the continuous n^{th} derivative $x^n(t_0)^2$ exists in a neighbourhood of a point t, then the uniform n^{th} difference derivative $x^n(t)$ also exists.
6. Let $T \epsilon B(X, X)$, where X is a Banach space. Prove that if $\| T \| < 1$, then $(I - T)^{-1}$ exists as a bounded linear operator on the whole space X and $(I - T)^{-1} = \Sigma T^j = I + T + \cdots$ where the series on the right is convergent in the norm on $B(X,X)$.
7. Prove that residual spectrum $\sigma(T)$ of a bounded self adjoint linear operator $T: H \to H$ on a complex Hilbert space H is empty.

SECTION -B

Answer any three questions **3×20=60**

8. Prove that for a set M of a metric space X to be compact, it is necessary that there exists finite ε-net for the set M for every $\varepsilon > 0$. Also prove that if the space X is complete, then this condition is sufficient.
9. Show that x^{-1} exists if and only if x is in no ideal
10. Prove that the resolvent set $\rho(T)$ of bounded linear operator T on a complex Banach space X is open, hence the spectrum $\sigma(T)$ is closed.
11. Prove that the n^{th} partial derivative of the function f exists in a neighbourhood of the point $T_\circ$ and if this derivative is continuous at $T_\circ$, then the n^{th} partial difference derivative also exists at $T_\circ$. Furthermore, both the derivatives coincide.
12. Prove that if X and Y are normed spaces. To each $T \epsilon B(X, Y)$ corresponds a unique $T^* \epsilon B(Y^*, X^*)$ that satisfies $<Tx, y^*> = <x\, T^*, y^*>$ for all $x \epsilon X$ and all $y^* \epsilon Y^*$. Moreover, T^* satisfies $\| T^* \| = \| T \|$.