STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015–16)

SUBJECT CODE : 15MT/RC/AA105

M.Phil. DEGREE EXAMINATION, JANUARY 2016
MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : **ADVANCED ALGEBRA AND ANALYSIS**
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ANY FIVE QUESTIONS: (5 x 8= 40)

1. Define the following terms:
   (i) Partially ordered set (ii) Totally ordered set (iii) Lattice
   (iv) Distributive lattice and (v) modular lattice
2. Define a Artinian ring and a Noetherian ring and give examples of a ring R such that
   (i) R is Noetherian but not Artinian (ii) R is Artinian but not Noetherian
   (iii) R is both Artinian and Noetherian.
3. Prove that $Z_m \otimes_z Z_n \cong Z_d$, as additive groups, where m and n are positive integers and$d$ is the gcd (m, n).
4. Define the following terms in a topological space X (i) Closure of a set E in X
   (ii) Compact set (iii) Hausdorff space (iv) Locally compact space
5. In a topological vector space X, prove that every convex neighborhood of 0contains a balanced convex neighborhood of 0.
6. State and prove Holder's inequality.
7. If $f \in L^1$, prove that $\hat{f} \in C_0$ and $\left\|\hat{f}\right\|_\infty = \|f\|_1$.

SECTION B

ANSWER ANY THREE QUESTIONS: (3 x 20= 60)

8. State and prove the fundamental theorem of projective geometry.
9. Let R be a left Artinian ring with unity and no non-zero nilpotent ideals. Then prove that R isisomorphic to a finite direct sum of matrix rings over division rings.
10. State and prove Riesz Representation theorem.

11. If X is a topological vector space with countable local base, then prove that there is a metric d on X such that

    (i) d is compatible with the topology of X

    (ii) the open balls centered at 0 are balanced

    (iii) d is invariant: $d(x+z, y+z) = d(x, y)$, for $x, y, z \in X$.

    If, in addition, X is locally convex, then d can be chosen so as to satisfy (i), (ii), (iii), and also(iv) all open balls are convex.

12. Prove that $L^p(\mu)$ is a complete metric space, for $1 \leq p \leq \infty$ and for every positive measure μ.

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