STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015-16)

SUBJECT CODE : 15MT/PC/MI24

M. Sc. DEGREE EXAMINATION, APRIL 2016
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : CORE
PAPER : MEASURE THEORY AND INTEGRATION
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

Answer allthe questions: **5×2=10**

1. Define measurable set.
2. What is a Borel set? Give an example.
3. Define Lebesgue integral of a function f.
4. When is a function convex on an open interval?
5. Define signed measure.

SECTION – B

Answer any fivequestions: **5×6=30**

6. Prove that outer measure of an interval is its length.
7. Let $\{E_i\}$ be a sequence of measurable sets. Show that
   (i) countable intersection of $\{E_i\}$ (ii) limsup(E_i),liminf(E_i) and lim$(E_i)$are measurable.
8. If f and g are real valued measurable functions defined on the same set E, prove that $f + g$, fg are measurable.
9. State and prove Fatou's lemma.
10. Prove that every function convex on an open interval is continuous.
11. State and prove Holders inequality.
12. Show that countable union of positive sets is positive.

SECTION – C

Answer any three questions: **3×20=60**

13. (a) Show that there exists a non measurable set.
    (b) Give an example to show that there exists an uncountable set of measure zero.

[12 + 8]

14. Prove that $M_e \neq P(\mathbb{R})$ and $\mathfrak{B} \neq M_e$ where
    M_e- the class of measurable sets
    $\mathfrak{B}$- the class of Borel sets and
    $P(\mathbb{R})$ - Power set of $\mathbb{R}$

15. State and prove Lebesgue dominated convergence theorem.

16. Show that $L^p(\mu)$ is a complete metric space for $1 \leq p \leq \infty$. [10 + 10]

17. (a) State and prove Radon Nikodym Theorem.

    (b) Show that the theorem is true for signed measure also. [15 + 5]