STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011–12& thereafter)

SUBJECT CODE: 11MT/PC/PD44

M. Sc. DEGREE EXAMINATION, APRIL 2016
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : PARTIAL DIFFERENTIAL EQUATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: (5 x 2 = 10)

1. Formulate the partial differential equation by eliminating the arbitrary function f from $z = x + y + f(xy)$.
2. Show that the partial differential equation $(n-1)^2 \frac{\partial^2 z}{\partial x^2} - y^{2n} \frac{\partial^2 z}{\partial y^2} = ny^{2n-1} \frac{\partial z}{\partial y}$ is of hyperbolic type.
3. Write Laplace equation in spherical coordinates.
4. State Neumann condition and insulated boundary condition.
5. What are the assumptions made in derivation of one-dimensional wave equation?

SECTION – B

ANSWER ANY FIVE QUESTIONS: (5 x 6 = 30)

6. Find the equation of the integral surface of the differential equation.

   $2y(z-3)p + (2x - z)q = y(2x-3)$, which passes through the circle.

   $z = 0, x^2 + y^2 = 2x$.
7. Show that the equations $xp = yq, z(xp + yq) = 2xy$ are compatible.
8. If $z = f(x^2 - y) + g(x^2 + y)$, where f and g are arbitrary functions, then prove that $\frac{\partial^2 z}{\partial x^2} - \frac{1}{x}\frac{\partial z}{\partial x} = 4x^2 \frac{\partial^2 z}{\partial y^2}$.
9. Derive Poisson equation.
10. Find various possible solutions of the one-dimensional heat conduction equation $\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$.
11. Derive Telephone equation.
12. Find D'Alebert's solution of one-dimensional wave equation.

SECTION – C

ANSWER ANY THREE QUESTIONS: **(3 x 20 = 60)**

13. (a) Find the general solution of $z(xp - yq) = y^2 - x^2$.

(b) Determine the characteristics of the equation $z = p^2 - q^2$ and find the integral surfacewhich passes through the parabola $4z + x^2 = 0$ and $y = 0$.

14. (a) Reduce the partial differential equation $y^2\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$ tocanonical form and hence solve it.

(b) If $\alpha_r D + \beta_r D' + \gamma_r$ is a factor of $F(D, D')$ and $\varphi_r(\xi\ )$ is an arbitrary function of the singlevariable ξ, then show that $u_r = exp\left(\frac{-\gamma_r x}{\alpha_r}\right)\varphi_r(\beta x - \alpha_r y)$ for $\alpha_r \neq 0$.

15. (a)Find the solution of the Laplace equation $\nabla^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

(b) State the interior Neumann problem for a circle and find its solution.

16. (a) Find the solution of the one-dimensional diffusion equation satisfying the following conditions: (i) T is bounded as $t \to \infty$ (ii) $\frac{\partial T}{\partial x}\big|_{x=0} = 0$,for all t (iii) $\frac{\partial T}{\partial x}\big|_{x=a} = 0$, for all t (iv) $T(x, 0) = x(a - x)$,$0 < x < a$.

(b) Find the temperature in a sphere of radius 'a' when its surface is maintained at zero temperature and its initial temperature is $f(r, \theta)$.

17. (a) Obtain the solution of the Radio equation $\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 v}{\partial t^2}$ appropriate to the case when a periodic e.m.f. V_0cospt is applied at the end $x = 0$ of the line.

(b) Reduce the one-dimensional wave equation $u_{tt} = c^2 u_{xx}$ to canonical form and find its solution.