STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011–12& thereafter)

SUBJECT CODE: 11MT/PC/FA44

M. Sc. DEGREE EXAMINATION, APRIL 2016
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : FUNCTIONAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION—A (5x2=10)

ANSWER ALL THE QUESTIONS

1. Define a bounded linear transformation.
2. Show that every non-zero Hilbert space possesses an orthonormal set.
3. Show that a normal operator need not be self-adjoint.
4. If x is an eigen vector of T then prove that x cannot correspond to more than one eigen value of T.
5. When do we say an element of a Banach algebra as topological divisor of zero?

SECTION—B (5x6=30)

ANSWER ANY FIVE QUESTIONS

6. Prove that a linear transformation T defined on a normed linear space N is bounded if and only if it is continuous.
7. State and prove closed graph theorem.
8. Prove that a closed convex subset C of a Hilbert space H contains unique vector of smallest norm.
9. If M is a closed linear subspace of a Hilbert Space H then prove that $H = M \oplus M^{\perp}$.
10. Prove that$P$ is the projection on a closed linear space M of H if and only if$(I - P)$ is a projection on $M^{\perp}$.
11. State and prove any three properties of spectrum of an operator on a finite dimensional Hilbert space$H$.
12. Prove that the set G of all regular elements of a Banach algebra is an open set.

SECTION—C (3x20=60)

ANSWER ANY THREE QUESTIONS

13. State and Prove the Hahn-Banach Theorem.
14. State and Prove Bessel's inequality.
15. a) Let H be a Hilbert space and T^* be adjoint of the operator T. Then prove that T^* is a bounded linear transformation and T determines T^* uniquely.

    b) Prove that an operator T on a Hilbert space H is unitary if and only if it is an isometric isomorphism of Honto itself.
16. If T is an arbitrary operator on a finite dimensional Hilbert space H, then prove that the spectrum of T is a finite subset of the complex plane and the number of points in spectrum of T does not exceed the dimension n of H.
17. a) Prove that the mapping $x \longrightarrow x^{-1}$ from the set of all regular elements G into itself of a Banach algebra is a homeomorphism.

    b) Define topological divisors of zero and prove the properties satisfied by them.