STELLA MARIS COLLEGE (AUTONOMOUS)
CHENNAI-86.
(For the academic year 2011-2012 and there after)

Subject code: 11MT/PC/DG44

M.Sc DEGREE EXAMINATION
Branch-I Mathematics
Fourth Semester

CORE: MAJOR CORE MARKS: 100
PAPER:DIFFERENTIAL GEOMETRY TIME: 3 HRS

## SECTION-A

Answer all the questions(5X2=10marks)

1. Define regular point.
2. Define diffeomorphism maps.
3. Write the first fundamental form of σ.
4. Define principal curvature.
5. Define Gaussian and Mean curvature.

## SECTION-B

Answer any five questions(5X6=30marks)

6. Prove that any reparametrisation of a regular curve is regular.
7. If $u$ and $\tilde{u}$ be open subsets of $R^2$ and let $\sigma: u \to R^3$ is a regular surface patch and if $\emptyset: \tilde{u} \to u$ be a bijective smooth map with smooth inverse map $\emptyset^{-1}: u \to \tilde{u}$, then prove that $\tilde{\sigma} = \sigma \circ \emptyset : \tilde{u} \to R^3$ is a regular surface patch.
8. Show that $\|\sigma_u X \sigma_v\| = (EG - F^2)^{\frac{1}{2}}$
9. State and prove Meusnier's theorem.
10. If $\sigma(u, v)$ is a surface patch with first and second fundamental forms $Edu^2 + 2Fdudv + Gdv^2$ and $Ldu^2 + 2Mdudv + Ndv^2$ respectively, then show that $K = \frac{LN-M^2}{EG-F^2}$ .
11. Find $\kappa$ for the circular helix $(a\cos\theta, a\sin\theta, b\theta)$.

12. Show that any tangent developable is isometric to a plane.

## SECTION-C

Answer any three questions(3X20=60marks)

13. If $\gamma(t)$ is a regular curve in $R^3$ with nowhere vanishing curvature and if $\frac{d}{dt}$ is denoted by a dot, then prove that $\tau = \frac{(\dot{\vartheta} X \ddot{\vartheta}).\dddot{\gamma}}{\|\dot{\gamma} X \ddot{\gamma}\|^2}$

14. If $\sigma: u \to R^3$ be a patch of a surface $S$ containing a point $P$ of $S$, and if $(u, v)$ be coordinates in $u$, show that the tangent space to $S$ at $P$ is the vector subspace of $R^3$ spanned by the vectors $\sigma_u$ and $\sigma_v$ .

15. Prove that the area of a surface patch is unchanged by reparametrisation.

16. State and prove Euler's theorem.

17. With the usual notation prove that

$$e_u^{'} e_v^{''} - e_u^{''} e_v^{'} = \lambda^{'} \mu^{''} - \lambda^{''} \mu^{'}$$
$$= \alpha_v - \beta_u$$
$$= \frac{LN - M^2}{(EG - F^2)^{\frac{1}{2}}}$$

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