STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2008–09)

SUBJECT CODE : MT/ME/FM54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2010
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – ELECTIVE
PAPER : FINANCIAL MATHEMATICS
TIME : 3 HOURS **MAX. MARKS : 100**

Answer Any Six Questions (each carrying 17marks)

I a. State central limit theorem.

b. Prove that the Geometric Brownian motion is a limit of simpler models.

c. An individual who plans to retire in 20 years has decided to put an amount A in the bank at the beginning of each of the next 240 months, after which she will withdraw \$1000 at the beginning of each of the following 360 months. Assuming a nominal yearly interest rate of 6% compounded monthly, how large does A need to be?

II a. Define spot interest rate.

b. Starting at some fixed time, let $S(n)$ denote the price of a certain security at the end of n additional weeks, $n \geq 1$. A popular model for the evolution of these prices assumes that the price ratios $S(n)/S(n-1)$ for $n \geq 1$ are independent and identically distributed lognormal random variables. Assuming this model, with lognormal parameters $\mu = .0165$ and $\sigma = .073$, what is the probability that (i) the price of the security increases over each of the next two weeks, and (ii) the price at the end of two weeks is higher than it is today? (Given $\emptyset(.226) = .5894$ and $\emptyset(.3196) = .6254$)

c. Suppose that you are to receive payments (in thousands of dollars) at the end of each of the next five years. Which of the following three payment sequences is preferable if the interest rate compounded annually at 10%?

A : 12, 14, 16, 18, 20

B : 16, 16, 15, 15, 15

C : 20, 16, 14, 12, 10

III a. State and prove the Law of One Price and illustrate the same by an example.

b. State and prove the put-call option parity formula.

IV a. Let $C(K,t)$ be the cost of a call option on a specified security that has strike price K and expiration time t. Prove that for fixed expiration time t, $C(K,t)$ is a convex and non-increasing function of K.

b. State and prove Arbitrage Theorem.

V a. Derive the Black-Scholes option cost, $C = e^{-rt}E[(S(t)-K)^{+}]$.

b. With usual notation, prove that $e^{-rt}E[IS(t)] = s\phi(\omega)$.

VI State and prove properties of the Black-Scholes option cost.

VII a. Prove that log utility function maximizes the long-term rate of return.

b. Suppose you are thinking about investing your fortune of 100 in two securities whose rates of return have the following expected values and standard deviations: $r_1 = 0.15$, $v_1 = 0.2$, $r_2 = 0.18$, $v_2 = 0.25$. If the correlation between the rates of return is $\rho = -0.4$, find the optimal portfolio when employing the utility function $U(x) = 1 - e^{-0.005x}$.

VIII a. Obtain 'value at risk' and 'conditional value at risk' for normal random variable.

b. Write a short note on Barrier Options.