STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2008 – 09)

SUBJECT CODE : MT/MC/RA54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2010
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : REAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

ANSWER ANY SIX QUESTIONS

1. a) Prove that $\lim_{x\to 1}\sqrt{x+3}=2$
   b) Let f be a nondecreasing function on the bounded open interval (a,b) . If f is bounded above on (a,b) then prove that $\lim_{x\to b^-} f(x)$ exists. (10+7)

2. a) If f and g are real valued functions, if f is continuous at a, g is continuous at $f(a)$ then prove that gof is continuous at a.
   b) Prove that the real valued function f is continuous at $a \in R'$ if and only if whenever $\{x_n\}_{n=1}^{\infty}$ is a sequence of real numbers converging to a then the sequence $\{f(x_n)\}_{n=1}^{\infty}$ converges to $f(a)$. (9+8)

3. a) Prove that in an Euclidean space $\mathbb{R}^k$ every cauchy sequence is convergent.
   b) Prove that in any metric space (S,d) every compact subset T is complete. (9+8)

4. a) Let $f: S \to T$ be a function from one metric space (S, d_S) to another (T, d_T). Then prove that f is continuous on S if and only if for every open set Y in T, the inverse image $f^{-1}(Y)$ is open in S.
   b) Let $f: S \to T$ be a function from one metric space (S, d_S) to another (T, d_T). If f is continuous on a compact subset X of S then prove that $f(X)$ is a compact subset of T. Prove that in particular, $f(X)$ is closed and bounded in T. (9+8)

5. a) Define homeomorphism.
   b) Define connected set.
   c) State and prove Bolzano's theorem. (4+4+9)

6. a) Prove that a metric space S is connected if and only if every two valued function on S is constant.
   b) State and prove the fixed point theorem. (9+8)

7. a) Let f be a bounded function on $[a,b]$. Then prove that every upper sum for f is greater than or equal to every lower sum for f.
   b) State and prove the chain rule. (9+8)

8. a) State and prove Rolle's theorem.
   b) State and prove the first fundamental theorem of calculus.

(9+8)