STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2008 – 09 & thereafter)

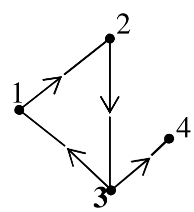
SUBJECT CODE : MT/MC/GT33

B. Sc. DEGREE EXAMINATION, NOVEMBER 2010
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR –CORE
PAPER : INTRODUCTION TO GRAPH THEORY
TIME : 2½ HOURS **MAX. MARKS : 100**

SECTION – A **(10 X 2 = 20)**
ANSWER ALL QUESTIONS

1. Give an example of complete graph, bigraph.
2. Define line induced sub graph with an example.
3. Define self complementary graph. Give an example.
4. Draw the graph $K_2 + K_3$.
5. Give an example of a closed walk of even length which does not contain a cycle.
6. Define cut point and bridge. Give an example for each.
7. Prove that if a graph G has exactly two points of odd degree, there must be a path joining these two points.
8. Exhibit a Hamilton cycle in the dodecahedron.
9. Draw all trees with five points.
10. State the nature of the connectivity of the digraph.



SECTION – B
ANSWER ANY FIVE OF THE FOLLOWING **(5x8=40)**

11. a) Prove that any self complementary graph has $4n$ or $4n+1$ points.
    b) Define the operations product and composition of the graphs G_1 and G_2 with an example.
12. If A is the adjacency matrix of a graph with $= \{v_1, v_2, \ldots, v_p\}$, prove that for any $n \geq 1$ the $(i,j)^{th}$ entry of A^n is the number of $v_i - v_j$ walks of length n in G.
13. a) Prove that a graph G with p points and $\delta \geq \frac{p-1}{2}$ is connected.
    b) If G is not connected, then prove that $\bar{G}$ is connected.
14. If G is a graph in which the degree of every point is at least two, then prove that G contains a cycle.
15. a) Prove that every connected graph has a spanning tree.
    b) Prove that every tree has a eentre consisting of either one point or two adjacent point.
16. State and prove Euler's theorem.

SECTION – C

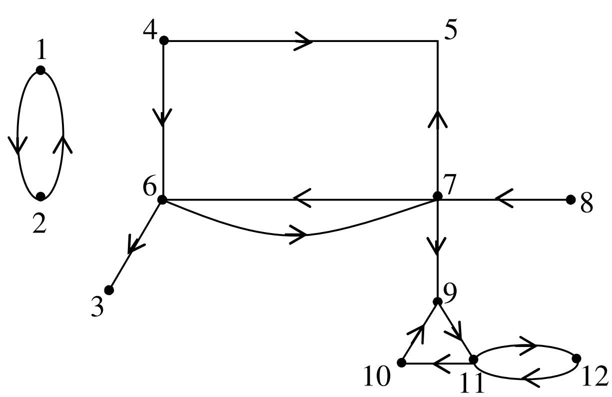
ANSWER ANY TWO OF THE FOLLOWING (2X20=40)

17. Define the following matrices of digraph and give an example for each.
    (i) Dominance matrix
    (ii) Reachability matrix
    (iii) Distance matrix
    (iv) Incidence matrix

18. a) Prove that a graph G with at least two points is bipartite if and only if al its cycles are of even length.
    b) If G is a graph with $p \geq 3$ points and $\delta \geq p/2$, then prove that G is Hamiltonian.

19. a) Let G be a $\{p, q\}$ graph. Prove that the following statements are equivalent.
    (i) G is a tree
    (ii) Every two points of G are joined by a unique path
    (iii) G is connected and $p = q + 1$
    (iv) G is acyclic and $p = q + 1$
    b) Prove that K_5 is non-planar. (15+5)

20. a) If two digraph are isomorphic, then prove that corresponding points have the same degree pair.
    b) Define condensation of a digraph. Find the condensation of the following digraph. (7+13)



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