**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086**
**(For candidates admitted during the academic year 2008–09 & thereafter)**

**SUBJECT CODE : MT/MC/DC12**

**B. Sc. DEGREE EXAMINATION, NOVEMBER 2010**
**BRANCH I - MATHEMATICS**
**FIRST SEMESTER**

**COURSE : MAJOR – CORE**
**PAPER : DIFFERENTIAL CALCULUS**
**TIME : 2½ HOURS** **MAX. MARKS : 100**

## SECTION – A (10 X 2 = 20)
## ANSWER ANY TEN QUESTIONS

1. Find the $n^{th}$ differential coefficient of $y = e^{ax}$.
2. Find $y_n$ if $y = \sin(ax + b)$.
3. Find $y_n$ if $y = \frac{1}{x^2+a^2}$.
4. Find $y_n$ if $y = cos^3x$.
5. If $u = \sin(ax + by + cz)$, find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}$.
6. If $u = (x^2 + y^2 + z^2)^{1/2}$, show that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 1$.
7. Find $\frac{du}{dx}$ when $u = x^2 + y^2$ where $y = \frac{1-x}{x}$.
8. Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3axy$.
9. Find the radius of curvature at the given point $(x, y)$ for the curve $xy = c^2$.
10. Write down the formula for the coordinates of centre of curvature.
11. Define an evolute.
12. Show that for the curve $s = ae^{x/c}$, $\rho = \frac{s}{c}\sqrt{s^2 - c^2}$.

## SECTION – B (4X20=80)
## ANSWER ANY FOUR QUESTIONS

13. a) (i) Find $y_n$ if $y = sin^3xcos^2x$.

    (ii) Find $y_n$ if $y = \frac{1}{x^2+5x+6}$.

    b) If $y = \sin(m\ sin^{-1}x)$ prove that $(1 - x^2)y_{n+2} = (2n + 1)xy_{n+1} + (n^2 - m^2)y_n$

14. a) If $y = e^{mcos^{-1}x}$, show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+!} - (n^2 + m^2)y_n = 0$

    b) If $y = a\cos(\log x) + bsin(\log x)$, show that

    $$x^2y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0.$$

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15. a) If $u = x^2(y-z) + y^2(z-x) + z^2(x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

    b) If $u = e^x(xcosy - ysiny)$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

    c) If $u = \log(\tan x + \tan y)$, prove that $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} = 2$.

16. a) Find the maximum and minimum of $x^2 e^x$.

    b) Discuss the maxima and minima of the function $x^3 y^2(6 - x - y)$.

17. a) Find the radius of curvature for the curve $\sqrt{x} + \sqrt{y} = 1$ at the point $\left(\frac{1}{4}, \frac{1}{4}\right)$.

    b) Show that for the parabola $y^2 = 4ax$, $\rho^2$ varies as $(SP)^3$, where $\rho$ is the radius of curvature at any point $P$ of the parabola and $S$ is the focus of the parabola.

    c) Find points on the parabola $y^2 = 8x$, at which the radius of curvature is $7\frac{13}{16}$.

18. a) Find the coordinates of the centre of curvature for any point $(x, y)$ on the parabola $y^2 = 4ax$. Also find the equation of the evolute of the parabola.

    b) In the curve $y = a\ log \sec\left(\frac{x}{a}\right)$, prove that the chord of curvature parallel to the axis of $y$ is of constant length.

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