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Endogenous Cycles in Rural to Urban Migration

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Abstract

We argue that the cost of rural–urban migration may act as an effective entry deterrence for successful migration to the urban sector. This cost results in some form of price rationing as the cash holding of a typical rural household falls short of the required cost. We demonstrate that certain endogenous forces will mesh in with the government policies to generate cumulative improvements in terms of trade in favour of the rural sector. Such improvements in terms of trade are shown to close the gap between the cost and the cash holding of a rural household. As a result, the rate of migration is positively related to the improvements in terms of trade in favour of agriculture. On the other hand, the terms of trade are shown to be inversely related to the rate of migration from agriculture. We establish, for the first time to the best of our understanding, that an endogenously driven and self-sustaining migration cycle would emerge from the rich dynamics involving migration flows and intersectoral terms of trade. We also demonstrates the possibility of a complex dynamics that can characterize the rural–urban migration and the attending development process. One can argue, on the basis of this type of complex dynamics, development process can be unpredictable and highly fragile. In other words, the principles of econophysics can offer an important new framework to understand labour flows in a complex society.

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Introduction

A somewhat unanticipated and, hence, a little embarrassing phenomenon of the modern time is a massive and unprecedented migration of people from rural to urban areas of developing nations. Todaro (1994) called this massive and unprecedented movement of people as 'the most perplexing dilemmas of the development process' (p. 14).

In this article, in order to explain 'the most perplexing dilemmas', we develop a model to establish that the dynamics of rural–urban migration, though based on very simplistic economic calculations, can engender chaotic behaviour, which is hitherto unknown, or unrecognized, in the literature on migration in development economics. The development of chaotic behaviour in development economics, by highlighting the regions of instability of the proposed model, can actually generate complex, yet deterministic, and bounded dynamics characterizing labour flows from rural to urban sectors of a developing economy. Chaos, in this context of migration, simply means that the values generated by the dynamic map, at pre-determined values of parameters of the proposed system, will become completely unpredictable. Unlike the common definition of chaos, the chaos described by the dynamical system of migration is the sheer unpredictability of labour flows arising from a deterministic structure, such as, a mathematical equation in Mitchell (2011). Our model thus shows that the migration process from rural to urban sectors can exhibit a hallmark of chaotic systems, namely, high sensitivity to initial conditions such that the deterministic system underlying migration can take a complex path that, under no circumstances, can be a priori predicted. Our model establishes that the rural–urban migration process may often resemble many known systems, such as, weather, biological populations and fluid turbulence, that are propelled by the chaotic behaviour.

It is manifestly obvious that the rates of rural to urban migration greatly exceed the capacity to employ them productively in urban industries of developing nations; yet such migration continues unabated and appears to be erratic. It is now understood that migration in excess of urban job opportunities represents both a symptom and a contributing factor to underdevelopment. The goal of our work is to highlight the endogenous forces at work that create cyclical paths in rural–urban migration, which can in turn generate extremely complex dynamics that the orthodox development economics has failed to capture. Let us take a cursory glance at the critical importance accorded to migration of factors in economic development.

In the broader framework of economic development, it is one of the most enduring realities that economic activities are unevenly distributed in space. The natural advantages of some regions over others usually lead to clusters of economic activities in more advantageous centres while others experience somewhat lower levels of economic activities. Should it surprise a keen observer of human development that nature is *divisive*? It should not since natural advantages are not equal among regions as different areas of our globe exhibit climatic advantages, relative resource abundance and different degrees of accessibility. All these factors, which are summarized under the much-celebrated label of *first nature*, play an important role in explaining economic concentration in specific locations, which is usually called *agglomeration* forces. In other words, the first nature sets some unequal constraints across space, which generates a spatial distribution of economic activities and unequal spatial (economic) development. It is also a part of our collective history that human beings have always reacted and responded to the binding constraints of the first nature, mainly to pursue their self-interests by taking advantages of the unequal (spatial) distribution of economic activities. The intended and unintended consequences of these human responses have acted upon and re-written the constraints of the first nature. Responses and reactions of economic agents to the constraints of the first nature form the core of the *second nature* that is motivated by the human zeal to 'truck, barter and trade' to make additional returns, which in turn incentivizes the *homo economicus* to try to lessen the tyranny of the first nature (see Gangopadhyay, 1997). Thus, within every region, there are forces that promote concentration of economic activities in the region, known as the centripetal force, which is in constant opposition with the centrifugal force that tends to disperse economic activities away from the region (see Gangopadhyay & Gangopadhyay, 2008).

The structure of a regional economy is influenced by the constant tensions between these twin forces. By modelling the sources of increasing returns to spatial concentration against the tendency to disperse, the New Economic Geography (NEG) teaches something extremely valuable about how and when these returns undergo changes and then examine how the regional economy's behaviour changes with them (see Gangopadhyay & Gangopadhyay, 2008). There are two important lessons from the basic findings of the NEG: first, it is widely held that the agriculture sector is rather a *misfit* for creating and driving agglomeration forces. Second, as the forces of agglomeration gather momentum the rural–urban divide will steadily rise with the rural sector lagging behind the urban sector. Both these views are in consonance with early models of economic development. Hirschman (1958, p. 183) pithily put forward the argument of unequal spatial development:

> We may take it for granted that economic progress does not appear everywhere at the same time and that once it has appeared powerful forces make for a spatial concentration of economic growth around the initial starting points.

Myrdal (1957, p. 26) sounded an early caution:

> The main idea I want to convey is that the play of the forces in the market normally tends to increase, rather than to decrease, the inequalities between regions.

A clearer dynamics was offered by Kaldor (1970, p. 340),

> As communication between different regions becomes more intensified (with improvements in transport and marketing organization), the region that is initially more developed industrially may gain from the progressive opening of trade at the expense of the less developed region whose development will be inhibited by it.

How serious can the urban problem be in developing nations? Only a generation ago large cities filled the urban landscape of advanced industrial nations. Today, developing nations home many mega-cities of our globe (see Krugman & Elizondo, 1996). To many observers, such urban sprawl in developing nations is a sheer economic disaster as Bairoch (1988) poignantly labelled these mega-cities as 'Romes without empires'. These cities are often a home ground for poverty, destitution and deprivation while the urban problem has been exacerbated by blatantly inadequate supply of local goods. There is little evidence that any perceptible improvement has taken place in the quality of life of nearly two billion people living in the urban sprawl (see Gangopadhyay & Nath, 1989, 2006, 2001a, 2001b; Gangopadhyay & Rahman, 2011; Gangopadhyay, 2014a). Urban crises continue unabated in developing nations after many decades of high rates of economic growth and significant social progress. In this work we model rural–urban migration as a dynamic process and seek to establish the following message: despite the fact that the migration dynamics are completely deterministic, yet we are to show that these dynamics can display complex cyclical fluctuations and even evolve in a chaotic fashion under a set of parametric restrictions. The importance dynamics in migration has been highlighted in several works, as Gunnar Myrdal (1944, p. 193) observed:

> Much in the Great Migration is left unexplained if we do not assume that there was before 1915 an existing and widening difference in living conditions between South and North which did not express itself to mass migration simply because the latter did not get a start and become a pattern.

The questions that have puzzled observers about the Great Migration in the US, and migration in general are as follows: if migrating to North from South was beneficial, why did not the Great Migration start before 1915? Why did migration display irregular path? Our model will offer some explanation of the endogenous dynamics involving migration. The desire to migrate is an age-old urge in human beings. Some forms of migration have been continuing since the Greko-Roman antiquity. Smith (1776) in his 'Wealth of Nations' addressed the motive behind migration and succinctly put forward:

> All the different states of ancient Greece possessed each of them, but a very small territory, and when the people in any of them multiplied beyond what the territory could easily maintain, a part of them were sent in quest of a new habitation in some remote and distant part of the world. (Book IV, CH VII, PT I, p. 58).

And thereby Smith (1776) unambiguously stressed the importance of a push factor behind an act of migration. When the home becomes too hot, migration to a new territory may reduce the economic hardship. At the same time Smith explained the pull factor behind migration since an act of migration is expected to bring additional fortune. Smith articulated this theme as,

> The people became clamorous to get land, and the rich and the great, we may believe were perfectly determined not to give them any part of theirs. To satisfy them in some measure, therefore, they frequently proposed to send out a new colony. (Book IV, CH VII, PT I, p. 59)

Smith, hence, clearly highlighted the desire to improve the material well-being of humans as a motivating force behind migration. The modern theory of rural–urban migration builds on this Smithian notion as the behavioural assumption posits that each migrant decides whether or not to make a move to city on the basis of implicit expected income maximization (see Todaro, 1971). Hence, the foundation stone of migration turns on the *postulate* of rational agency. There are two important constituents in such decision making: first, the wage differential between urban and rural sectors which acts as an attractor. Second, important element is the likelihood of finding a job in the urban sector that drives a decision maker to take plunge into the urban sector. This likelihood is usually captured in a probability estimate. As we know, an emphasis has been placed on the issue of rural–urban wage differential as a determining factor behind rural–urban migration (see Beals, 1967; Harris & Todaro, 1968). Todaro (1971) summarizes the received doctrine as the following,

> If the migrant anticipates a relatively low probability of finding regular wage employment in the initial period but expects this probability to increase over time as he is able to broaden his urban contacts, then it would still be rational for him to migrate even though expected urban income during the initial period or periods might be lower than expected rural income (p. 393).

As a consequence the decision to migrate to an urban area hinges upon a long-run optimization scheme such that the present value of the expected income streams from urban employment exceeds the present value of the stream from rural incomes. After arriving in the urban sector, migrants search for a job till they are successful. The basic message of these models turns on the following intuition: a representative migrant pays a fixed fee (the cost of migration) to join the ranks of urban unemployed. After having joined the ranks of urban unemployed, he samples the urban job market looking for a job. Successive developments of the literature on migration theory focused attention on the search techniques of a migrant when he enters the urban market (see Alchian, 1970; Phelps, 1972; Stigler, 1961), tough the complexity of the rural society has largely been ignored as highlighted in Gangopadhyay (2014b, 2014c). As a result, the orthodox development economics ignored the relevance of social heat within the rural and urban societies and intolerances to migrants and conflicts and influences of conflicts on further migration (see Gangopadhyay, 2014c, 2015; Gangopadhyay & Gangopadhyay, 2007; Gangopadhyay et al., 2011).

Social heat in the context of migration calls forth a careful discussion: migration shrinks the divide of the urban world from the rural setting and thereby and puts various stocks of people of diverse backgrounds, cultures, ethnicities, languages, religions and possibly races in a close proximity (Gangopadhyay, 2009). In the absence of an arbiter and mediator, the progress and prosperity of urbanization have been accompanied by heightened risks of mutual intolerances and potential conflicts between different people. As a result, urbanization often takes place with ghettoization as highlighted by Gangopadhyay (2012) and the ghettoization leads to inter-group intolerances, social splintering and social incohesion that prevent effective and orderly interaction between human groups in an urban setting. One may call this disharmony between human groups as social heat. In other words, people from different stocks and/or religions live as a minority and thereby expose themselves to the dangers of intolerances of varying degrees by the majority in a community and vice versa. This is increasingly becoming a common feature of modern societies; especially in developing nations today. As explained by Gangopadhyay (2009), in Chapter XIII of *Leviathan* Hobbes suggests that men are quarrelsome by nature. If this is so, a large scale migration from rural to urban sectors can offer unprecedented opportunities to some groups for dumping their acts of intolerance on others. According to Hobbes there are three main causes of intolerances: first, the underlying theme of competition for limited resources motivates men to oppose others. Second, the mutual distrust induces men to engage in conflicts for safety. Finally, men lock horns for achieving glory—intolerance and conflict for reputation. It is the *social contract* and the enforcement of this social contract that are believed to control social heat within reasonable bounds in organized societies. However, if there is a problem in enforcing the social contract at a reasonably low cost—it is not possible to keep the lid on the Pandora's box (see Gupta, 2012). In simple terms, the idea of people about others is influenced by their simple and often naive impressions acquired from social interactions as highlighted by Hume (1888), which can often fail to generate any social capital between human groups living in the same society as highlighted by Durlauf (1999) among others.

In the context of humans the social contract is to prevent the vulnerable from being molested by the powerful (Rousseau, 1964). In *contrat social* Rousseau popularized this idea of the social contract, which is recognized as the major difference between the animal world and the world of humans. Here lurks Rousseau's famous paradox: in entering society man sacrifices all his rights, but really he gives up nothing (see Cobban, 1934; Hollis, 1987). Rousseau's solution is that man be both legislator and subject and undertake his *civil burden* most diligently to express the true interests of his society by voicing its 'general will'. His solution does not necessitate an enforcement of the social contract by an omnipotent and omniscient state since agents, driven by their civil duties, ensure its enforcement. Hobbes in Chapter XIII of *Leviathan* realizes that it is not an easy task to protect the vulnerable from the powerful in any society simply because the powerful will wilfully take on his civil burden. The Hobbesian suggestion is to create a 'common power' by the social contract 'to keep all in the awe'. It is widely recognized that there is a need to enforce the social contract by a legislative mandate. Wherever such a mandate is impossible, a society strives to tackle the enforcement problem by erecting customs and social norms that influence individual behaviours in the social context. Thus, what action a man chooses can be seriously influenced by existing social customs and norms. An example may be helpful: consider the wage bargaining problem as outlined in Akerlof (1980) in which union leaders are bound by their members' normative expectations to hold out against a management whose social position makes concessions equally unacceptable to their stakeholders. This idea of Akerlof is akin to the market in gifts that is governed by the norms of gift giving—what is appropriate to give and to whom and at which occasion. Typically these norms are cast in iron, which uniquely determine individual actions wherefrom a social outcome evolves—given a well-defined and enforceable penalty mechanism. The problem with intersectoral migration is the arrival of new ideas, new people and their novel cultures, norms and customs, and it is not sacrosanct that the old norms will still be applicable to the new people. It is even more problematic for the old people to accept the norms of the new people. The resultant social heat can stop an effective mixing of people to turn a group of people into a community (see Putnam, 1993).

Intergroup differences in terms of language, ethnicity, culture and religions are identified as potential sources of social heat, or group conflicts. Evidence shows that risks of such conflict rises as a relatively homogenous society gets fractured with new entrants. Beyond a point, social heat and conflict risks seem to subside as the number of groups increase as highlighted by Collier (2000) and Fearon and Laitin (2003). It is usually held that conflict risk is highest where there are a few large groups, which indicates an intermediate level of fractionalization of the society. The latent idea is that the demographic polarization index is higher the more nearly the society is divided into two equal sized groups (Montalvo & Reynal-Querol, 2005) while the formation of groups is based on ethnicities, languages, cultures or races. It is also shown that the demographic composition is combined with other factors like relative deprivation and horizontal inequalities to raise the risk of conflict (Estaban & Ray, 1999).

Our work is organized around a neglected, yet a very important, aspect of migration theories, namely, the cost of migration. Existing theories, though, stress the *rational* basis of migration decision but scarcely address the associated cost. Such cost is treated as an 'entry fee' while the implicit assumption is that a migrant can easily foot this bill.[1] Consequently, migration theories examine how a rational agent would best deploy the available means to achieve the goals/ends and also address relevant policy implications. We, on the contrary, go beyond this postulate of rational agency to highlight the *capabilities* of rural agents by focusing on the cost side of migration (see Sen, 1985).

To put it baldly, the migration cost is the price that a migrant pays for the 'urban life'. We lump the pecuniary and non-pecuniary (net) benefits from the urban sector under the term urban life. The desire to migrate hence embodies a demand for urban life. The thorny question is whether the demand constitutes an effective demand. If the demand fails to coincide with the effective demand, it is trivially true that a migration decision will fail to materialize. The net gain from migration may be positive and large, yet a rational agent would not migrate since he cannot afford to pay the price. There would hence emerge some kind of price rationing. The price rationing subsumes that the cost of migration is greater than the cash holdings of a rural household, which is exacerbated by an absence of capital markets.

The price of urban life may be quite exorbitant and can turn out to be very important for certain categories of migrants. Especially those migrants who need to establish their career paths in a specific manner will require a steady flow of cash over a long time. Say, for example, a mechanic from the rural sector who requires to get his degree recognized in the urban sector, which may involve a long time and a good deal of money. In less developed countries (hereafter LDCs) such costs may assume paramount importance. Once one looks carefully at costs of migration and successful migration, there is a deep philosophical issue at stake: Leibenstein (1957) expounded the idea that work capacity is directly linked to food intake. On this basis, Dasgupta and Ray (1987, 1986) rigorously demonstrated the *critical* link between involuntary unemployment in LDCS and the incidence of malnutrition that is, in turn, explained by the distribution of assets. As a result, if a migrant does not have a steady source of nutrition during the waiting period, he may be competed out by workers having better cash/economic position.

A successful migration may call forth an adequate cash holding/fund that will allow him to stay above the minimal nutritional level. Thus, following Dasgupta and Ray (1986, p. 1030), one may argue that a migrant without an adequate cash fund/flow is at a disadvantage in the urban labour market relative to those who posses this cash. Consequently, he may be caught up in the vicious cycle of malnutrition, unemployment and malnutrition. There is, hence, a reason to believe that cash disadvantage may result in employment barriers for migrants. And rational agents will carefully consider such barriers before making a decision. Consequently, the cost of migration will become an important and limiting factor behind a decision to migrate. In this work, we posit that a migrant must have a vector of endowments to have a successful footing in the urban sector and the pecuniary cost of this vector is christened as the cost of migration.

If a barrier to employment exists, then some rural migrants will confront an entry barrier if their cash holdings fail to cover the cost of migration. The main purpose of this work is to model a complex interrelationship between individual (migration) decisions and their economic constraints (cash holdings) wherefrom arises endogenous forces that drive the evolution of these decisions and economic constraints over time. We argue that endogenous forces will cause cumulative improvements in terms of trade (TOT, hereafter) in favour of agriculture that would gradually close the gap between the cost of migration and the cash holding of potential migrants. As the gap closes, the entry barrier disappears and the flow of migration significantly goes up. It is argued that such a migration involves a transfer of resources from rural to the urban areas and, hence, gradually puts a brake on the improvement in TOT. The upshot of the analysis is that the TOT and the migration flow mutually feed on each other. Migration flow is positively related to an improvement in TOT whereas the TOT is inversely associated with rural–urban migration. This work demonstrates that the TOT and the rate of migration act on each other in such a way that both these variables move in a complex fashion. The cost of migration, along with some specific endogenous forces, can engender endogenously driven and self-sustaining migration and TOT cycles. These cycles will move counter to the urban business cycle. We also establish that the time profile of migration and cash holdings can engender a very complex and even a chaotic path of rural–urban migration. The possibility of a chaotic rural–urban migration dynamics can render the process of development highly fragile.

The plan of the study is the following. The second section provides the rural backdrop and explains the endogenous changes in TOT that would reduce the (real) cost of migration. The third section analyzes the effects of changes in TOT on the cash holdings of rural households and demonstrates that there exists a finite improvement in TOT that would achieve the balance between migration cost and cash holdings of a typical rural household. The fourth section examines the endogenous forces that induce a cumulative improvement in TOT, which will establish this balance. This section also argues that an increase in the rate of migration adversely reacts on the TOT that will engender an endogenously driven and self-sustaining migration cycle. The fifth section expands the basic model to establish how a complex dynamics can characterize the time path of rural–urban migration. The sixth section offers concluding comments.

The Rural Set-up

There are two types of economic organization in the rural sector. First, there are farms that do not hire any labour inputs from the rural labour and run the agricultural production on the basis of family labour. Second, there are farms that organize production solely on the basis of hired labourers. We call the first type of farms as Farm I while the second type of farms is labelled as Farm II. We postulate that a Farm I has identical members and the payoff function of a member is the following:

$$Z(C/n, h) = U(C/n) - V(h) \tag{1}$$

U denotes the utility of a member from the consumption C and V represents the disutility from labour. C is the agricultural product that Farm I keeps for consumption. We assume C to be constant. And h is the working hours of each member of Farm I, n is the number of such members in a typical family farm. As h goes up, the disutility from labour $V(h)$ goes up. If total labour hours remain fixed, then as n falls h goes up which increases the utility from per capita consumption $U(C/n)$. Hence the following is true:

$$\frac{d(1/n)}{dh} = \alpha > 0 \tag{2}$$

The utility maximization yields the following:

$$\frac{U'(C/n)}{V'(h)} = \frac{1}{\alpha C} \tag{3}$$

We postulate the following:

$$V'(h) > 0,\ V''(h) > 0 \text{ and } U'(C/n) > 0,\ U''(C/n) < 0.$$

From these restrictions on the first and second derivatives one can ensure a unique vector (h*, (C/n)*).

The optimality condition $[U'(C/n)\alpha C]/V'(h) = 1$ is fulfilled at (h*, (C/n)*). Any deviation from (h*, (C/n)*) inflicts a loss of welfare on an average worker in Farm I. The point of departure of this work is the assumption that Farm I is characterized by surplus labour so that such a family farm can easily transfer family labour to the urban sector without adversely affecting output. The only constraint that binds such a family is the cash/liquidity constraint. A successful migration needs a particular amount of cash/investment fund I which poses a cash constraint on family farms.[2]

On the other hand, Farm II runs on a commercial basis and its constraint on migration is also the illiquidity. Hence if a family of Farm II acquires the necessary cash holding/fund, such families also send their members to the urban sector. The migration reduces the availability of labour in Farm II as Farm II does not have a pool of surplus labour. As a result, these families hire labour from Farm I families which would increase the rural wage rate. Farm II hence faces an additional cost from migration.

The Cost of Migration

The purpose of this sub-section is to show that the 'real' cost of migration and the improvements in TOT in favour of agriculture are inversely associated. To show such an inverse relationship we need some details which are the following: the cash holding/fund I is the cost that a migrant must pay for a successful migration. We write the fund (I) that a migrant needs to make a migration as the following:

$$I = p_U w_1 b_U + w_2 b_r p_r \tag{4}$$

When p_U is the price index of urban goods, p_r is the price index of rural goods, b_U is the vector/bundle of urban goods and b_r is the vector of rural goods which a migrant must be able to purchase in order to have a successful migration. Note that w_1 and w_2 are the weights attached to these vectors of urban and rural goods, respectively. These weights represent the entitlements of a migrant over these goods which would provide him a secure position in the urban labour market. This is so, since the household can without any cost (or, at a little cost) may periodically send some rural products to the migrant. Similarly, the migrant may receive entitlements of some urban products by doing casual jobs. Hence one may call this cash fund '*I*' as the start-up capital, or investment fund, to gain an access to successful urban 'life'.[3] Assuming b_U and b_r as composite goods, the cost, or fund *I*, in terms of the overall price index is given by:

$$R = \frac{I}{w_1 p_U + w_2 p_r} = \frac{w_1 b_U + w_2 b_r \dfrac{p_r}{p_U}}{w_1 + w_2 \dfrac{p_r}{p_U}} \tag{5a}$$

Let us label the improvement in TOT as X such that

$$X = \frac{d\left(\dfrac{p_r}{p_U}\right)}{dt} \tag{5b}$$

The overall price index P is a weighted average of p_r and p_U while for the sake of simplification we assume same weights, w_1 and w_2. We call R as the real cost of migration.

Observation 1: The real cost/investment fund, R, depends critically on the intersectoral terms of trade, p_r/p_U, while w_1, w_2, b_r and b_U are the structural parameters.

Proposition 1: As the terms of trade improves in favour of agriculture, the real cost R declines if $b_U > b_r$.

Proof: Totally differentiating (5b) with respect to (p_r/p_U) we get the following:

$$\frac{dR}{dX} = \frac{w_1 w_2 (b_r - b_U)}{[w_1 + w_2 X]^2} \tag{6}$$

Since $b_r < b_U$ and they are composite commodities,

$$\frac{dR}{dX} < 0$$

Equation (6) establishes the inverse relationship between the real cost of migration and the improvement in TOT in favour of agriculture. Hence one may linearize the relationship between R and the improvement in terms of trade as the following:

$$R = N_1 - N_2 X \tag{6a}$$

for $N_1 > 0$, $N_2 > 0$.[4] Equation (6a) subsumes that an improvement in TOT reduces the real cost by N_2 per cent. And hence the gap between the cost and the cash holdings declines. In the next section we show how an improvement in TOT increases the cash holdings of rural households, leading to a further decline in the gap.

Evolution of Cash Holdings

The purpose of this section is to show that there exists a finite improvement in TOT that endows rural households with the required cash fund that will result in a successful migration from the rural to the urban sector.

Proposition 2: As the terms of trade improves in favour of agriculture, the value of surplus, S, of Farm I goes up.

Proof: If q is the output produced of Farm I, then the value of surplus S is given by the following:

$$S = (1 - r)\, q\, p_r/p_U \tag{7}$$

Where r is the household farm's fixed propensity to consume. It is easy to check that, ceteris paribus, an improvement in terms of trade increases the nominal surplus.

Observation 2: For family farm of type II, the time-profile of cash holdings is the following:

$$S_t = S_{t-1} + (1 - r)\, q\, d(p_r/p_U)/dt \tag{7a}$$

$$= S_{t-1} + (1 - r)\, qX \tag{7b}$$

For a large farm q may be so large that a small improvement in terms of trade would induce a high cash effect such that:

$$S_t > R \tag{7c}$$

These families send their members to urban sector as they now acquire the required liquidity. It will result in an increase in the demand for labour which would increase rural employment as well as rural wage rate. Hence the wage earnings of Farm I would go up. The increase in wage earnings of Farm I is wLX: an increase wage earnings of Farm I following an improvement in terms of trade. By postulating a linear and increasing relation between L and X and setting the elasticity of demand for labour with respect to X equal to one, we arrive at the above figure for an increase in wage earnings.

Proposition 3: There exists a finite improvement in the terms of trade, X^*, such that for any $X > X^*$, the accumulated cash of Farm I will exceed the required cash fund I.

Proof: The cash balance of Farm I is what follows:

$$M_t = (1 - r)\, q\, X + wLX + M_{t-1} \tag{8}$$

M_t is the cash balance at date t, M_{t-1} is the initial cash balance and the first term on the right side of Equation (8) represents an increase in the cash holding due to an improvement in terms of trade (Equation 7a). The second term labels the increase in cash holding due to an increase in wage earnings.

Combining Equations (8) and (6a) we derive X^* as the following:

$$\left(\frac{d(p_r/p_U)}{dt}\right)^* = X^* = \frac{N_1 - M_{t-1}}{(1-r)q + wL + N_2} > 0 \tag{9}$$

This threshold improvement in terms of trade is positive if the initial cash balance is less than the initial cost of migration before any change in terms of trade occurs. As a result, if actual terms of trade improves beyond this threshold level, successful migration can take place from Farm I.

The point of departure of this section is to highlight that the cost of migration imposes price rationing on some rural households which acts as an effective entry deterrence. Section two and section three show that improvements in TOT gradually close the gap between the cost and cash holdings. It is further shown that there exists a finite improvement in TOT which completely closes the gap and hence enables rural agents to migrate. The crucial question is whether the policy regimes and the market forces can mesh in to generate a cumulation of improvements in TOT. This cumulation leads to the disappearance of the entry barrier. We address this possibility in the next section.

Migration Cycle

In this section we give our attention on the time profile of improvement in terms of TOT and the time profile of labour outflow from the rural sector. Let us denote the improvement in terms of trade in favour of agriculture by X while Y describes the labour outflow. At any point of time t, $X(t)$ conditions on the excess demand for the agricultural good. Hence, we write

$$X(t) = kE(t) \tag{10a}$$

Equation (10a) is the age-old excess demand function where $E(t)$ labels the excess demand for the agriculture product k is an arbitrary speed of price adjustment. Consider a typical competitive equilibrium with uneven distribution of assets in a LDC (see Dasgupta & Ray, 1986). In such an equilibrium, it is possible that the asset-less suffer involuntary unemployment due to their low consumption intakes (Dasgupta & Ray, 1986, 1987). The obvious policy implication is to aim for more egalitarian distribution of food. Consequently, an activist government may get involved in food transfers and accumulation (see Dasgupta & Ray, 1987). These policies will result in an excess demand for food that will increase the TOT in favour of agriculture. It is possible that the improvement in TOT may instead result in a further increase in the excess demand for food/rural products. And hence there would be a cumulative improvement in TOT in favour of agriculture. This is so for three sets of reasons: first of all, as the TOT improves in favour of agriculture, many agents will face food entitlement failure (see Koopmans, 1957; Sen, 1985). The government, hence, will be under pressure to acquire more food to meet the consumption requirements of people. This will in turn increase the excess demand for food/rural products. Second, an improvement in TOT causes a transfer of resources from urban to rural areas which will also boost the overall demand in the rural economy. Such an increase in rural income will result in an increase in the demand for food/rural products. Finally, the substitution effect in this context may be too weak due to the nature of goods produced in the rural sector. The cumulative improvement in TOT endow rural households with increasing cash holdings that will result in increasing migration once the rural economy gathers sufficient cash holdings.

The excess demand in agriculture sector depends upon two things: first the intersectoral terms of trade and second on the income of the agricultural households. Just consider a single household. As the intersectoral terms of trade moves in favour of agriculture, the industrial urban good becomes relatively cheaper and hence a substitution effect lowers the excess demand for agricultural/rural goods. On the other hand, such a movement in terms of trade increases the households income which in turn act positively on excess demand for the agricultural goods. We assume that the income effect of such a price change to be stronger than the overall substitution effect. On top of that, since more people suffer food entitlement failures in urban areas, the government must buy more agricultural goods. As a result if the TOT go up in favour of agriculture, it simultaneously increases the excess demand for agricultural goods which further increases the price of the agricultural goods in terms of the industrial/urban good. Based on this intuition we provide the following proposition.

Proposition 4: If $\dot{X}$ is the rate of the improvement of terms of trade in favour of agriculture, then the time path of $\dot{X}(t)$ can be approximated by the following differential equation in which $Y(t)$ is the migration flow:

$$\frac{\dot{X}(t)}{X(t)} = a - bY(t) \tag{10b}$$

Proof: Assuming k to be a constant, Equation (10a) provides us the following:

$$\dot{X}(t) = ke(t) \tag{11a}$$

While $e(t)$ is the change in excess demand (E) for agricultural goods. We argued that the excess demand for agricultural goods is an increasing function of the improvement in TOT for the three reasons discussed earlier. Hence we assume the following:

$$e(t) = F(X(t)) \tag{12a}$$

where $FN > 0$

Linearizing F we yield the following:

$$e(t) = \alpha_1 X(t) \tag{12b}$$

While α_1, >0, measures the sensitivity of excess demand with respect to a change in intersectoral TOT. There are two distinct impacts on the excess demand, $E(t)$, as the TOT improves. First, such an improvement increases the excess demand for rural goods due to the three reasons discussed earlier. It is crucial to note that the impact of $X(t)$ on excess demand, on the contrary, declines as migration whisks off a part of the cash holdings to the urban sector. This flight of cash holdings effectively does two things: first, this flight depresses the available net cash holdings/economic resources of the rural sector which results in a decline in the demand for rural goods. This immediately lowers the excess demand $E(t)$. As a result this elasticity α_1 would be adversely affected by the migration flow $Y(t)$. One can easily set up the following:

$$\alpha_1 = (\beta - \theta Y(t)) \tag{12c}$$

Without any migration as the TOT improves by 1 per cent, the excess demand goes up by α_1 per cent which is equal to β per cent. Once the migration starts, economic resources move from the rural to the urban sector which reduces the impact of X on E. And hence as the TOT improves by 1 per cent, the excess demand for the rural goods increases by $(\alpha_1 - \theta Y)$ per cent. We assume that the decline in excess demand is an increasing and linear function of $Y(t)$ and we normalize the relationship by setting the relevant sensitivity coefficient equal to one. Substituting (12c) in (12b) and substituting (12b) in (12a) we get the following:

$$\dot{X}(t) = k(\beta - \theta Y(T))\, X(t) \tag{13}$$

Writing $\beta k = a$, and $b = \theta k$ we derive the following equation:

$$\frac{\dot{X}(t)}{X(t)} = a - bY(t) \tag{10b}$$

The time path of the intersectoral TOT is approximated by the earlier differential Equation (10b). Equation (10b) purports that the TOT improves at a rate 'a' until the migration flow $Y(t)$ transfers a substantial amount of resources from the rural to the urban sector. Such a transfer reduces the pressure on the excess demand for rural goods which in turn puts a downward pressure on the TOT. Now we turn to the rate of migration.

Proposition 5: The time path of migration from rural to urban sector follows the differential equation:

$$\frac{\dot{Y}(t)}{Y(t)} = -c + dX(t) \tag{14a}$$

Where $\dot{Y}(t)$ is the change in migration flow.

Proof: This proposition has the fulcrum mainly on two intuitions. First, since the migration needs a cash balance the rate of migration ($Y(t)$) slows down as the cash balance of rural households dwindles. Hence if $Y(t)$ is current migration, it reduces the current cash balance of rural households, ceteris paribus. As the current cash balance declines, future migration $\dot{Y}(t)$ also declines. Hence, ceteris paribus,

$$\dot{Y}(t) = -cY(t) \tag{14b}$$

Second important element is the positive effect of improvement in intersectoral terms of trade on the cash balance which gives a boost to migration. Hence, $X(t)$ acts positively on $Y(t)$. We write this as:

$$\dot{Y}(t) = -cY(t) + fX(t) \tag{14c}$$

for a constant $f > 0$

The coefficient 'f' captures the effect of $X(t)$ on the change in migration flow. We further postulate that coefficient 'f' depends on the successful migration as the urban family members would provide a boost to rural urban migration. Hence we write:

$$f = dY(t) \tag{14d}$$

for a constant $d > 0$

The effect of $X(t)$ on $\dot{Y}(t)$ is higher, the higher is the initial value of $Y(t)$ since larger the number of members from rural households established in urban areas, the higher is the subsequent increase in migration. Combining (14a), (14b), (14c) and (14d) we arrive at the following:

$$\frac{\dot{Y}(t)}{Y(t)} = -c + dX(t) \tag{14a}$$

As agents move from rural households to the urban sector, their cash holdings decline which will put a brake on further migration. Such an adverse effect of migration is captured by the coefficient c. On the contrary, there are two positive impacts on the rate of migration. First of all, as the TOT goes up, the cash holdings increase which enable rural households to migrate. And second, as rural agents move to the city, due to greater volume of urban 'connection' migration rate would go up. These two positive effects are captured by the term 'd'. The upshot of propositions 4 and 5 is that the time paths of migration and intertemporal terms of trade engender a special type of differential equation system which is the much acclaimed Lotka-Volterra case of prey–predator. The natural developments of the system are contained in the following two observations:

Observation 3: The equilibrium rates of change in migration flow and change in terms of trade are given by the following:

$$g_1{}^* = \dot{Y}(t)/Y(t) = c/d \quad (16a)$$

$$g_2{}^* = \dot{X}(t/X(t) = a/b \quad (16b)$$

Proof: See Franke (1988) for the derivation of the above.

Observation 4: The solution trajectory $S(X, Y)$ of the differential Equation (10b) and (14a) lies in a closed positive orbit in the (X, Y) space.

Proof: This is the well-known property of the Lotka-Volterra differential equations system as pressed into action by Goodwin (1967). The equilibrium rates are well defined at $g_1{}^*$, $g_2{}^*$, but the actual system will never converge to the equilibrium rates. The migration rates and inter-sectoral terms of trade would gyrate around $g_1{}^*$, $g_2{}^*$. Goodwin (1967) observes, 'One initial condition selects the curve, a second fixes the starting point, then we traverse along some particular curve B in the direction of the arrows forever, in the absence of given outside changes' (Goodwin, 1967, p. 169).

Let us try to elucidate the cyclical movements. Consider the following diagram (Figure 1).

The direction of arrows is arbitrary and so is the solution path that are determined by the initial conditions. When the terms of trade increase is X_{min}, migration rate Y is average, or equilibrium rate $g_2{}^*$ (OB). As the terms of trade further improves, the increase in cash holdings pushes up the migration rate which reaches the maximum Y_{max} (OA). At Y_{max}, the flight of cash to urban areas depresses the rural demand, vis-à-vis increase in urban demand, so much that the increase in terms of trade start slowing down and hence X declines. As X declines, migration rate also slows down which gradually stabilizes the demand for rural goods. As gradually the demand for rural goods picks up, the TOT improves in favour of agriculture which in turn improves the cash holdings of the rural households. As the cash holdings build through time migration rate Y picks up and hence continues the cycle.

Observation 5: The earlier cyclical fluctuations in rates of migration and TOT movements arise because of the special characteristics of

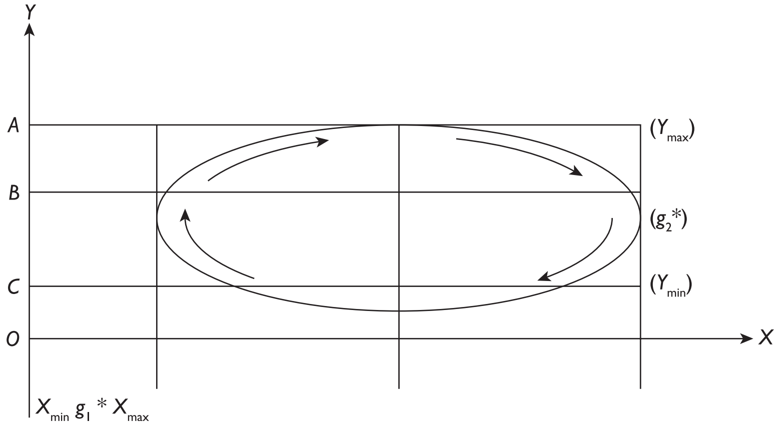

Figure 1. Cyclical Fluctuation in Migration and TOT

Note: OA = Y_{max}, OB = g_2*, OC = Y_{min}.

the excess demand function for agricultural goods. This excess demand function is shown to be non-monotonic due to the impacts of migration and transfer of resources from rural to urban areas and government commitments in food markets on the excess demand for agricultural goods. In other words, the income effect is shown to dominate the substitution effect for agricultural products for certain values of the intersectoral TOT. In future work, it may be desirable to provide a complete microfoundation to this type of excess demand function.

Complex Dynamics in Migration

We begin with a modified version of Equation (14a):

$$\dot{Y}(t) = [-c + dX(t)]Y(t) + \delta \quad (14a)$$

Note δ is an exogenous rate growth in migration that is independent of accumulation of cash balances. Applying Equations (14a)–(14d) and taking discrete values of change in X and Y we get the following difference Equation from (11a):

$$Y_{t+1} = (1 - c)Y_t + dX_tY_t + \delta \quad (17a)$$

We use time subscript to represent the difference equation, as opposed to differential equations of prevision sections. As argued before, we postulate that X, the terms of trade, is an increasing function of the migration flow (Y):

$$X_t = \phi Y_t \tag{17c}$$

where $\phi > 0$.

It is noteworthy that for deriving our results we only need ϕ to be non-zero and the sign does not matter. From the aforementioned we get the following dynamics.

Lemma 1: The migration dynamics and the terms of trade dynamics can be captured by the following difference equation of X:

$$Y_{t+1} = \gamma_3 + \gamma_1 Y_t + \gamma_2 Y_t^2 \tag{17d}$$

where

$$\gamma_1 = (1 - c) \tag{17e}$$

$$\gamma_2 = (d\phi) \tag{17f}$$

$$\gamma_3 = \delta \tag{17g}$$

Proof: The derivation, being simple, is omitted. Details are available in Gangopadhyay (2005). QED.

Lemma 2: The aforementioned dynamics (17d) has two fixed points Y^*, Y^{**}:

$$Y^* = [(1 - \gamma_1) - \text{SQRT}\{(1 - \gamma_1)^2 - 4\gamma_2\gamma_3\}]/(2\gamma_2) \tag{18a}$$

$$Y^{**} = [(1 - \gamma_1) + \text{SQRT}\{(1 - \gamma_1)^2 - 4\gamma_2\gamma_3\}]/(2\gamma_2) \tag{18b}$$

Y^{**} is always unstable. Y^* is stable if

$$\text{SQRT}\{(1 - \gamma_1)^2 - 4\gamma_2\gamma_3 < 2 \tag{18c}$$

Proof: The derivation, being simple, is omitted. Details are available in Gangopadhyay (2005). QED.

If Y^* is stable, then the terms of trade dynamics (17d) will drive the migration flow to the equilibrium Y^* if the initial migration rate (flow) is close enough as dictated by (18c). If the migration flow at any date t should go beyond the threshold Y^{**}, then this unstable fixed point will cause the migration flow to diverge to infinity. Therefore, for the migration flow to be bounded it is imperative that the following is true:

$$Y_t < Y^{**} = Y^{\max} \text{ for } t = 0, 1, 2, 3.... \text{ and} \tag{18d}$$

$$Y_t > [Y^{**} - (\gamma_1/\gamma_2)] = Y^{\min} \text{ for } t = 0, 1, 2, 3.... \tag{18e}$$

Thus, the migration flow will be bounded if the initial migration lies within the interval $[Y^{\min}, Y^{\max}]$ and

$$\text{SQRT}\{(1 - \gamma_1)^2 - 4\gamma_2\gamma_3 < 3 \tag{18f}$$

If the restrictions on the parameters and the initial migration flow, Equations (18d)–(18f) hold the migration dynamics remain bounded between $Y^{\min}$ and $Y^{\max}$. Following Feigenbaum (1978) we now apply the change of variable technique that will transform the non-linear migration dynamics to the logistic equation of May (1976).

Lemma 3: The quadratic migration dynamics (17a) is equivalent to the following logistic equation with an appropriate transformation of the variable *Y*:

$$Z_{t+1} = \gamma_3(Y^{**} - Y_t)/A \tag{19a}$$

$$A = 1 + \text{SQRT}\{(1 - \gamma_1)^2 - 4\gamma_3\gamma 1\} \tag{19b}$$

$$Z_{t+1} = AZ_t(1 - Z_t) \tag{19c}$$

Proof: The derivation is omitted. QED.

For $1 < A < 3$ the dynamics of migration converges to the stable equilibrium rate Y^*. If $A > 3$ then Y^* becomes unstable and the migration flow converges to a stable two-period cycle. As A is increased further the stable period cycles of period n bifurcates into cycles of $2n$. At $A = 3.57$ the migration flow evolves through a cycle of infinite period. The migration flows are within the relevant bounds but they never repeat. For a higher order, the migration flow may look like a random process but they are fully deterministic. For values of A greater than 3.57 we can have even more complex behaviour.

Result 1: The migration flows will evolve through a cycle of infinite period and hence never repeat themselves if

$$\phi\delta > [2.57^2 - a^2]/4b \tag{20a}$$

The main problem with the existing line of neoclassical research in development economics has been its sole reliance on the equilibrium analysis as a tool of investigation. However, migration from rural to urban areas, changes in TOT and transfer of resources from rural to urban areas belong to the domain of economic dynamics. Issues concerning migration thus embrace a very dynamic field that is, in turn, influenced by bubbles of expectations, intense desires for 'urban life' and a constant quest for survival of a family by having a foot in urban centres. It thus seems that the equilibrium approach to modelling migration ignores various important facets of this arena of research. One may argue this as a general weakness of the neoclassical development economics. Neoclassical development economists typically focus their attention on economic models with regions of local stability on the assumption that regions of instability are of little importance and more of a pathological case as highlighted in econophysics (see Gangopadhyay, 1997, 1999, 2005, 2007, 2012; Gangopadhyay et al., 2011, 2014). The main justification for using the equilibrium analysis is that development economics does not find exploding time paths of any significant variable. This justification is incorrect once we introduce the possibility chaotic dynamics. The development of chaotic behaviour significantly undermines this dismissal of regions of instability that can actually generate complex, yet deterministic, dynamics within bounds. This is where we pitch our work to highlight the importance of chaotic behaviour in the context of development economics.

Concluding Comments

Desire to migrate and the rationality behind migration are well explored issues. Yet modern theories have not examined the implications of the cost of migration. This cost represents the price a migrant must pay for the 'urban life'. As a result, there may emerge a possibility of price rationing. Despite a potential income/utility gain from migration an agent may not have 'capability' to migrate since he cannot pay the price. Such cost hence acts as a barrier to entry. This work shows how endogenous forces may mesh in with government policies to generate a steady and cumulative improvement in terms of trade in favour of the rural sector. Such cumulative improvements tend to close the gap between the cost of migration and the cash holdings of rural households and the price rationing will gradually disappear. The rate of rural–urban migration will pick up. Since migration entails a transfer of cash and resources from the rural to the urban sector, it will adversely impinge on the TOT. Thus, the flow of migration and the change in TOT are shown to feed on each other. We argue, on the basis of these feedbacks, that there would emerge endogenously driven and self-sustaining migration cycle that runs counter to the urban business cycle. This work also demonstrates the possibility of a complex dynamics that can characterize the rural–urban migration and the attending development process. One can argue, on the basis of this type of complex dynamics, development process can display significant fragility.

The finding has important messages for neoclassical development economics built on the postulate of rational agency: it is typically assumed in the deductive equilibrium approach of neoclassical development theory that the Nash-Walras equilibrium can dispel all systematic prediction errors and an economic system will settle in an equilibrium characterized by self-confirming and mutual-best responses. The deductive equilibrium analysis may have contributed to the understanding of modern development economics by focussing its attention on the region of stability. However, little attention has been given to the regions of instability. We, upon examining the region of instability, establish that the postulated rural–urban dynamics can exhibit chaotic behaviour. Economic actors (be farmers, governments, economists or industrialists) now fail to see systematic errors. Economic agents can fail to make long run predictions with certainty even though they act in a purely deterministic world. Time profiles, which start very close together, will separate exponentially. The strength of deductive equilibrium gets terribly emasculated in the context of development economics. We conclude that an application of standard results of chaotic behaviour in the context of rural–urban migration can be a very important step forward to understand the dynamics of economic development.

Appendix

In what follows we offer an introduction to the concepts of the logistic map function and its application to complex systems and chaos theory. The most common equation for the logistic map for a time-varying variable P is as follows:

$$P_{(t+1)} = R(P_t - P_t^2/k) \qquad \text{(A1)}$$

Where the variable *Pt* at a given time step *t* determines the variable at the next time step $P_{(t+1)}$ in the sequence. The value *R* is the rate at which the variable changes over time and the constant k is the maximal value of the variable P. In the context of population studies k is the 'carrying capacity', or the number of individual organisms, that the organism's environment can support.

The aforementioned equation can be rewritten in the following simple form:

$$Q_{(t+1)} = R\,Q_t\,(1 - Q_t) \tag{A2}$$

Where the value Q is the ratio of P and k, which is the population given as a percentage of the total carrying capacity in the biological science (see Mitchell, 2011). This Equation (A2) is the most common form of the logistic equation. The logistic map gives a mathematical equation that, despite its simplicity, can give rise to chaotic behaviour. Chaos signifies that the values generated by the logistic map, at a given R-value, will become unpredictable. The logistic map summarizes a chaotic system that is high sensitivity to initial conditions. Scientists usually create data sets by simply varying the input parameter R for Equation (A2), the initial value of Q and the number of time steps to be generated. The actual function is simply given by (A2). The bifurcation property explains the behaviour of the logistic map at a large time value threshold: the software script, mostly in Matlab, executes the logistic map function until the threshold is reached, then the software samples the next one hundred data points. If the logistic map has reached a stable state (cycling through a set of values) then the resulting data points contains less than the sample amount of unique values. If the logistic map has not reached stability, all of the data points are unique and represents chaotic behaviour. If one plots the earlier map, the plot will show the areas of stability and instability over values of R, including the negative region. The positive region of the map shows the stability and chaotic behaviour of the logistic map for the various values of R. R, in the positive case, represents a growth regime for the variable Q. When R is negative the variable Q is stable at 0 between $-1 < R < 0$, then for $R < -1$ the population stability splits and ranges between values $-0.5 < Q < 1.5$. What is important for scientists is the self-similarity. Each plot can split (from 1 to 2, 2 to 4, 4 to 8) has a similar branch. For Equation (A2) each split happens a rate of ~4.669 as fast as the previous split, which is christened as the Feigenbaum's constant (see Feigenbaum, 1978). Another important property of the earlier equation is the region of stability of the dynamic path (A2) between $3.8 < R < 3.9$. The time path, or the behaviour of Q, the logistic map suddenly becomes stable at two values. These values split in a similar way as the stable regions below ~3.569 and proceed to fall into another chaotic region at about $R > 3.87$. The earlier behaviour is due to the attractors acquiring a steady state with two values. The pattern repeats with a 'stair step', corresponding with additional bifurcation points. When the chaos value for R is reached the plot of the dynamic Equation (A2) becomes correspondingly chaotic. While in the chaotic region the probability of any value repeating would appear to be zero. The fact that there are a finite number of time steps in the calculation means there is an apparent upper bound on the uncertainty. Of note, in the chaotic region, is the appearance of regions that have a set of non-increasing, finite uncertainty. These regions correspond to the stable regions, which often appear periodically. Note also that the width of the regions of stable uncertainty, prior to the chaotic boundary, can be shown to follow the ratio outlined by Figenbaum's constant.

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Notes

1. One exception is the work by Carrington et al. (1996) that introduced cost of migration that is endogenously driven by the volume of migration. In our work, on the contrary, we consider the impact of endogenous economic factors on the purchasing power of potential migrants.
2. Because of the surplus labour, it is easy to check that a family members support such a migration decision. For an intuitive appeal, if h^A is the per head labour requirement to produce C units of family consumption which gives each member $(C/n)^A$ of consumption. Now with migration h^A tends towards h^* and $(C/n)^A$ tends towards $(C/n)^*$ which would increase the welfare of the remaining family members. As a result, migration may be welfare improving.
3. Please note that we are playing down the importance of the probability of finding a job just to reduce the complexity of the analysis. Such an exogenous probability would not materially alter the analysis.
4. Note that the weighting scheme in the price index may well differ from the weighting scheme in the investment fund. A different weighting scheme would call for a condition under which equation (6a) is correct.

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