STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2008 – 09)

SUBJECT CODE : MT/PE/TS34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2009
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : ELECTIVES
PAPER : TENSOR ANALYSIS AND SPECIAL THEORY OF RELATIVITY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**

ANSWER ANY FIVE QUESTIONS

1. a) If ϕ is an invariant, determine whether $\frac{\partial^2 \phi}{\partial x^p \partial x^q}$ is a tensor.

   b) Show that $\frac{\partial A_p}{\partial x^q}$ is not a tensor even though A_p is a covariant tensor of rank one.

2. State and prove the Quotient law of tensors.

3. Define fundamental metric tensor g_{ij} and the conjugate tensor g^{ij} and show that they are tensors of rank two.

4. Prove that $\nabla^2 \phi = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^p} \left(\sqrt{g}\ g^{pq} \frac{\partial \phi}{\partial x^q} \right)$ and hence obtain $\nabla^2 \phi$ in spherical polar coordinates.

5. Explain the experiment of Michelson and Morley to determine the motion of the earth with respect to the privileged frame of reference in which the spend of light was to be uniform.

6. Define aberration of light and obtain the classical and relativistic formulae for it.

7. Explain the relative character of simultaneity of events and obtain the relativistic law of addition of velocities.

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SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. a) Show that the geodesics in a Riemannian space are given by

$$\frac{d^2x^r}{ds^2}+\left\{\begin{matrix} r \\ p\ \ q \end{matrix}\right\}\frac{dx^p}{ds}\frac{dx^q}{ds}=0.$$

b) Prove that the covariant derivatives of g_{ij}, g^{ij} and δ^i_j are zero.

9. a) Derive the transformation laws for the Christoffel's symbols of the first and the second kinds.

b) Evaluate the Christoffel's symbols of the fist and the second kinds for spaces where $g_{pq}=0$ if $p\neq q$.

10. Define Galilean transformation equations and show that the laws of mechanics are covariant with respect to these equations.

11. Obtain Lorentz transformation equations and discuss in detail the effect of the Lorentz equations or length and time measurements in different frames of reference.

12. Obtain the formula for the relativistic mass of a moving body by considering the elastic collision of two identical mass points. Also deduce the formula for relativistic kinetic energy of a mass point.