STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2008 – 09)

SUBJECT CODE : MT/PC/TO35

M. Sc. DEGREE EXAMINATION, NOVEMBER 2009
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – CORE
PAPER : TOPOLOGY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**

ANSWER ANY FIVE QUESTIONS

1. State and prove Cantor intersection theorem.
2. If X is an infinite set; T consists of the empty set together with all subsets of X whose complements are finite. Prove that (X,T) is a topological space.
3. If A is a subset of a topological space X, prove that
   (i) $\overline{A} = A \cup D(A)$ (ii) A is closed iff $A \supseteq D(A)$.
4. State and prove the Heine-borel theorem.
5. Prove that every sequentally compact metric space is totally bounded,
6. Prove that a one-to-one continuous mapping of a compact space onto a Hausdorff space is a homoeomorphism.
7. Prove that in a locally connected space X each component is open.

SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. a) Prove that every separable metric space is second countable.
   b) Prove that any subspace Y of a complete metric space X is complete iff it is closed.

9. a) State and prove Tychonoff's theorem.
   b) Define Bolzano Weierstrass property. Prove that every compact metric space has the Bolzano Weierstrass property.

10. a) Define a normal space. Prove that every compact Hausdorff space is normal.

b) If X is a normal space; A and B are disjoint closed subspaces of X. Prove that there exists a continuous real valued function of defined on all of X, taking values in the closed interval $[a,b]$ of the real line such that $f(A)=a$ and $f(B)=b$.

11. State and prove Urysohn imbedding theorem.

12. a) Prove that X is disconnected $\Leftrightarrow$ there exists a continuous mapping of X onto the discrete a two point space $\{0,1\}$.

b) Prove that a compact Hausdorff space is totally disconnected iff it has an open base whose sets are also closed.

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