STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2009 – 10)

SUBJECT CODE : MT/PC/RA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2009
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR – CORE
PAPER : REAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**

ANSWER ANY FIVE QUESTIONS

1. Prove that union of any collection of open sets in $\mathbb{R}^n$ is an open set in $\mathbb{R}^n$ and intersection of finite collection of open sets in $\mathbb{R}^n$ is an open set in $\mathbb{R}^n$. Show by an example that the intersection of any collection of open sets in $\mathbb{R}^1$ need not be open in $\mathbb{R}^1$.

2. Let (M,d) be a metric space. Define $d_1(x,y)=\dfrac{d(x,y)}{1+d(x,y)}$ where $x,y\in M$. Prove that d_1 is also a metric on M.

3. Show that the sequence of real valued functions $\{f_n(x)\}$, where $f_n(x)=\dfrac{1}{1+nx}$ where $0<x<1$ and $n=1,2,...$ converges pointwise but not uniformly on $(0,1)$. Also show that $g_n(x)=\dfrac{x}{1+nx}$ where $0<x<1$ and $n=1,2,...$ converges uniformly on $(0,1)$.

4. Derive Bessel's inequality and hence deduce Parseval's formula.

5. State and prove Riesz-Fischer theorem.

6. a) Let S be subset of $\mathbb{R}^n$ and $\bar{f}:S\to\mathbb{R}^n$. Define directional derivative of $\bar{f}$ at $\bar{c}$ in the direction $\hat{u}$. Derive the directional derivative of a linear function $\bar{f}:\mathbb{R}^n\to\mathbb{R}^n$ at $\bar{c}$ along $\hat{u}$.
   b) If $\bar{f}$ is differentiable at $\bar{c}$. Prove that $\bar{f}$ is continues at $\bar{c}$.

7. Let A be an open subset of $\mathbb{R}^n$ and assume that $\bar{f}:A\to\mathbb{R}^n$ has continues partial derivatives $D_i f_i$ on A. If $\bar{J}_{\bar{f}}(\bar{x})\neq 0$ for all $\bar{x}$ in A, prove that $\bar{f}$ is an open mapping.

SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. a) State and prove Cantor's intersection theorem.

   b) Define open covering for a set S. Show that every open covering of a subset of $A \subseteq \mathbb{R}^n$ contains a countable subcovering . (10+10)

9. a) Assume that $f_n \to f$ uniformly on $S \subseteq \mathbb{R}^1$. If each f_n is continuous at a point c of S then prove that the limit function f is also continuous at c.

   b) State and prove Cauchy condition for uniform convergence of sequence of functions (8+12)

10. a) State and prove Jordan's theorem

    b) State and prove Riemann Lebesgue lemma (10+10)

11. a) State and prove Chain rule for derivatives in $\mathbb{R}^n$.

    b) State and prove Mean Value Theorem. (12+8)

12. State and prove Inverse function Theorem.