STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086 (For candidates admitted during the academic year 2009 – 10)

SUBJECT CODE: MT/PC/ME14

 $(5 \times 8 = 40)$

M. Sc. DEGREE EXAMINATION, NOVEMBER 2009 BRANCH I - MATHEMATICS FIRST SEMESTER

COURSE : MAJOR - CORE PAPER : MECHANICS

TIME : 3 HOURS MAX. MARKS : 100

SECTION – A

ANSWER ANY FIVE QUESTIONS

- 1. Show that the total angular momentum of a system about a point *O* is the angular momentum of the system concentrated at the center of mass, plus the angular momentum of motion about the center of mass. Prove also that the kinetic energy of the system consists of two similar parts.
- 2. Using Calculus of Variations, find the curve joining tow points, along which a particle falling from rest under the influence of gravity travels from the higher to the lower point in the least time.
- 3. Define Coriolis force and use it to explain the eastward deviation from the vertical of a freely falling particle.
- 4. State and prove Euler's theorem on the motion of a rigid body with one point fixed.
- 5. Define Legendre transformation and hence derive Hamilton's equations of motion
- 6. Define Lagrange and Poisson brackets and show that they are canonical invariants.
- 7. Show that the transformation $Q = \log(I/q \sin p)$, $P = q \cot p$ is a canonical transformation and that the function which generates this transformation is $F_3 = e^{-Q} \cos p$.

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SECTION - B

 $(3 \times 20 = 60)$

ANSWER ANY THREE QUESTIONS

- 8. a) Derive Lagrange's equation for holonomic, conservative system.
 - b) Discuss the problem of Atwood's machine using Lagrange's formulation.
- 9. a) Derive Lagrange's equations for non-holonomic systems.
 - b) Illustrate the use of Lagrange multiplier method by considering the example of a loop, rolling without slipping, down an inclined plane.
- 10. Define Euler angles to describe the configuration of a rigid body with one point fixed and obtain the transformation matrix A connecting the body set and space set of axes in terms of these angles.
- 11. State and prove the principle of Least Action explaining clearly the symbols used and the assumptions made.
- 12. Obtain the necessary and sufficient condition for a set of transformation of coordinates to be canonical in the simplectic notation, by considering both the cases when the transformation is time independent and time dependent.