STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2009 – 10)

SUBJECT CODE : MT/PC/MA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2009
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR – CORE
PAPER : MODERN ALGEBRA
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**
ANSWER ANY FIVE QUESTIONS

1. Prove that the number of elements conjugate to a in G (a group) is the index of the normalizer of a in G.
2. If p is a prime and p^m divides $o(G)$, p^{m+1} does not divide $o(G)$ then prove that G has a subgroup of order p^m.
3. If A and B are finite subgroups of G. Then prove that
$$o(A \times B) = \frac{o(A)\ o(B)}{o(A \cap (xBx^{-1}))}$$
4. Suppose that G is the internal direct product of $N_1, N_2 \ldots\ldots, N_k$. Then prove that (i) for $i \neq j$ $N_i \cap N_j = \{e\}$ and (ii) $a \varepsilon N_i$, $b \varepsilon N_j$ implies $ab=ba$
5. State and prove Division algorithm for polynomials.
6. If L is finite extension of K and if K is finite extension of F prove that L is finite extension of F.
7. Prove that for every prime number p and every positive integer m there exists a field having p^m elements.

SECTION – B **(3 X 20 = 60)**
ANSWER ANY THREE QUESTIONS

8. State and prove Sylow's Theorem.
9. (a). Prove that a polynomial of degree n over a field can have atmost n roots in any extension field.
   (b). If F is of characteristic 0 and if a and b are algebraic over F, then prove that there exists an element c in $F(a,b)$ such that $F(a,b) = F(c)$.
10. State and prove the Fundamental Theorem of Galoi's Theory.
11. State and prove fundamental theorem on finitely generated modules.
12. State and prove Wedderburn's Theorem on finite division rings.