STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2015–16)

SUBJECT CODE : 15MT/MC/AT14

B. Sc. DEGREE EXAMINATION, NOVEMBER 2015
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR – CORE
PAPER : ALGEBRA AND TRIGONOMETRY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10X2=20)**
ANSWER ALL THE QUESTIONS

1. Solve the equation $x^3 - 12x^2 + 39x - 28 = 0$ whose roots are in A.P.
2. If the roots of $x^3 - 12x^2 + 23x + 36 = 0$ are $-1,4,9$ find the equation whose roots are $1, -4, -9$.
3. Increase by 7 the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$.
4. Determine completely the nature of the roots of the equation $2x^5 - x^3 + 10x - 8 = 0$.
5. Prove that product of two orthogonal matrices is orthogonal.
6. Find the eigen values of $\begin{pmatrix} 1 & 2 & 0 \\ 4 & 3 & 0 \\ 5 & 6 & 7 \end{pmatrix}$.
7. Express $\cos 5\theta$ interms of $\cos\theta$.
8. Prove that $cosh^2x - sinh^2x = 1$.
9. Prove that $sinh^{-1}x = \log(x + \sqrt{x^2 + 1}$.
10. Find $\log 3$.

SECTION – B **(5X8=40)**
ANSWER ANY FIVE QUESTIONS

11. If α, β, γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$ form the equation whose roots are $\alpha\beta, \beta\gamma$ and $\gamma\alpha$.
12. Solve $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.
13. If a, b, c be the roots of the equation $x^3 + px^2 + qx + r = 0$ find the equation whose roots are $bc - a^2,\ ca - b^2,\ ab - c^2$.
14. Verify Cayley-Hamilton theorem for $\begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$.
15. Separate into real and imaginary parts of $\tan(x + iy)$.
16. Prove that $\frac{\sin 7\theta}{\sin\theta} = 7 - 56sin^2\theta + 112sin^4\theta - 64sin^6\theta$.
17. Separate the real and imaginary parts of $(\alpha + i\beta)^{x+iy}$.

SECTION – C **(2X20=40)**
ANSWER ANY TWO QUESTIONS

18. a) Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$ whose roots are in harmonic progression.

b) Find the positive root of the equation $x^3 - 2x^2 - 3x - 4 = 0$ correct to three places of decimals by Horner's method.

19. a) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$.

b) Solve $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$.

20. a) Prove that $cos^5\theta\ sin^7\theta = \frac{-1}{2^{11}}[\sin 12\theta + 2\sin 10\theta - 4\sin 8\theta + 10\sin 6\theta + 5\sin 4\theta - 20\sin 2\theta]$

b) If $\tan h\frac{u}{2} = \tan\frac{\theta}{2}$ prove that $u = \log\ \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$.

c) If $\tan[\log(x + iy)] = a + ib$ then prove that
$\tan(\log(x^2 + y^2)) = \frac{2a}{1-a^2-b^2}$, $a^2 + b^2 \neq 0$

(8+7+5)