STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2011–12 & thereafter)

SUBJECT CODE: 11MT/MC/VA34

B. Sc. DEGREE EXAMINATION, NOVEMBER 2015
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – CORE
PAPER : VECTOR ANALYSIS AND ITS APPLICATIONS
TIME : 3 HOURS MAX. MARKS : 100

SECTION-A

Answer All the questions (10 x 2 = 20)

1. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$ then prove that $\nabla r^n = nr^{n-2}\,\vec{r}$.
2. Find the unit normal vector to surface $x^2 + 3y^2 + 2z^2 = 6$ at the point (2,0,1).
3. Prove that curl $\vec{r} = 0$.
4. Show that the vector $3x^2y\vec{i} - 4xy^2\vec{j} + 2xyz\vec{k}$ is solenoidal.
5. If $\vec{F} = xy^2\vec{i} + 2x^2yz\vec{j} - 3yz^2\vec{k}$, find curl $\vec{F}$ at the point $(1, -1, 1)$.
6. If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, evaluate $\int_c \vec{F}\,.d\vec{r}$ where c is the curve on the xy – plane $y = 2x^2$ from (0,0) to (1,2).
7. State Frenet - Secret Formulae.
8. Give the physical significance of div and curl of a vector point function.
9. State Stoke's theorem.
10. Show that $\iint_s \vec{r}\,.\vec{n}\,ds = 3V$ where V is the volume enclosed by the closed surface s.

SECTION-B

Answer any FIVE questions (5 x 8 = 40)

11. A particle moves along the curve $x = t^3 + 1, y = t^2, z = 2t + 5$ where t is the time. Find the components of its velocity and acceleration $t = 1$ in the direction $\vec{i} + \vec{j} + 3\vec{k}$
12. Find the directional derivative of the function $\varphi = xy + yz + zx$ in the direction of the vector $2\vec{i} + 3\vec{j} + 6\vec{k}$ at the point (3,1,2).
13. Derive the vector identity, $\nabla.(\vec{u} \times \vec{v}) = \vec{v}\,.(\nabla \times \vec{u}) - \vec{u}\,.(\nabla \times \vec{v})$.
14. Show that surface $5x^2 - 2yz - 9x = 0$ and $4x^2y + z^3 - 4 = 0$ are orthogonal at (1, -1, 2).

15. Find $\int_C \vec{F}\,.\,d\vec{r}$ where $\vec{F} = (x^2 - y^2)i + 2xy\vec{j}$ and c is the square bounded by the co-ordinate axes and the lines $x = a$ and $y = a$.

16. Using divergence theorem, evaluate $\iint_s \vec{F}\,.\vec{n}\;ds$ where $\vec{F} = 4xz\vec{i} - y\vec{j} + yz\vec{k}$ and s is the surface of the cube bounded by the planes
$x = 0, x = 2, y = 0, y = 2, z = 0, z = 2.$

17. Find the area of the circle using Green theorem.

SECTION-C

Answer any TWO questions **(2 x 20 = 40)**

18. (a) Find the value of 'a' such that $\vec{F} = (axy - z^2)i + (x^2 + 2yz)\vec{j} + (y^2 - axz)\vec{k}$ is irrotational.

    (b) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$ show that $\nabla.(r^n\,\vec{r}\,) = (n+3)r^n$.

    (c) Find the equation of the tangent plane and normal to the surface $xyz = 4$ at the point $(1, 2, 2)$. (4 + 8 + 8)

19. (a) Evaluate $\iint_s \vec{F}.\vec{n}\;ds$ where $\vec{F} = 18Z\vec{i} - 12\vec{j} + 3Y\vec{k}$ and s is the part of the plane $2x + 3y + 6z = 12$ which is located in the first quadrant.

    (b) Verify Green's theorem in the plane for $\int_c (xy - y^2)dx + x^2\;dy$ where 'c' is the curve of the region bounded by $y = x$ and $y = x^2$ (10 + 10)

20. (a) Verify Stoke's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where s is the half surface of the sphere $x^2 + y^2 + z^2 = 1$ and c its boundary .

    (b) State and prove Gauss divergence theorem. (10 + 10)