**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**
**(For candidates admitted during the academic year 2011 – 12 & thereafter)**

**SUBJECT CODE : 11MT/MC/RA54**

**B. Sc. DEGREE EXAMINATION, NOVEMBER 2015**
**BRANCH I - MATHEMATICS**
**FIFTH SEMESTER**

**COURSE : MAJOR – CORE**
**PAPER : REAL ANALYSIS**
**TIME : 3 HOURS** **MAX. MARKS : 100**

**SECTION – A (10X2=20)**
**ANSWER ALL THE QUESTIONS**

1. Define limit of a function on the real line.
2. Show that the function defined by $f(x) = \frac{\sin x}{x}$ , $x \neq 0$

   $$f(0) = 1$$

   is continuous at $x = 0$.
3. Define open subset of a metric space.
4. If $E$ is a subset of a metric space $M$ and if $x \in M$ is a limit point of $E$, prove that every open ball $B(x, r)$ contains atleast one point of $E$.
5. Prove that a sequence $\{x_n\}$ in a metric space $(s, d)$ can converge to atmost one point in S.
6. Define complete metric space and give an example.
7. Define disconnected metric space.
8. State intermediate value theorem for real continuous functions.
9. When do we say that $f$ is Riemann Integrable on $[a, b]$.
10. Show that differentiability at a point implies continuity at that point.

**SECTION – B (5X8=40)**
**ANSWER ANY FIVE QUESTIONS**

11. If $\lim_{x \to a} f(x) = L$ and if $\lim_{x \to a} g(x) = M$, show that $\lim_{x \to a} f(x) + g(x) = L + M$.
12. If $\mathcal{F}$ is any nonempty family of open subsets of a metric space $M$, show that $\bigcup_{G \in \mathcal{F}} G$ is also an open subset of $M$.
13. If $F_1$ and $F_2$ are closed subsets of the metric space show that $F_1 \cup F_2$ is closed.
14. In any metric space $(S, d)$, show that every compact subset $T$ is complete.

15. Show that a metric space $S$ is connected if and only if every two valued function on $S$ is constant.

16. If $f \in \Re[a,b]$ and $a < c < b$, show that $f \in \Re[a,c]$, $f \in \Re[c,b]$ and $\int_a^b f = \int_a^c f + \int_c^b f$.

17. State and prove the law of the mean.

**SECTION – C** **(2X20=40)**

**ANSWER ANY TWO QUESTIONS**

18. a) If $f$ is continuous at $a$, prove that $\lim_{n\to\alpha} x_n = a$ implies $\lim_{n\to\alpha} f(x) = f(a)$.

 b) If $G_1$ and $G_2$ are open subsets of the metric space $M$, prove that $G_1 \cap G_2$ is also open.

19. a) If $f$ is continuous on a compact subset $X$ of $S$, prove that the image $f(x)$ is a compact subset of $T$.

 b) State and prove fixed point theorem for contractions.

20. a) State and prove Rolle's theorem.

 b) State and prove the second fundamental theorem of calculus.