STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2011–12 & thereafter)

SUBJECT CODE : 11MT/MC/PT54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2015
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : PROBABILITY THEORY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10X2=20)**
ANSWER ALL THE QUESTIONS

1. Define probability.
2. State the multiplication theorem of probability.
3. Define probability mass function of a discrete random variable.
4. If $f(x) = \begin{cases} Kx(1-x) & 0 < x < 1 \\ 0 & elsewhere \end{cases}$

   is a pdf of a continuous random variable X find the value of K.
5. Show that $cov(aX, by) = ab\ cov(X, Y)$.
6. State uniqueness theorem of characteristic functions.
7. Write the recurrence relation for the moments of Binomial distribution.
8. Find the m.g.f. of Poisson distribution.
9. Mention any two characteristics of Normal Probability curve.
10. Write the points of inflection of Normal Curve.

SECTION – B **(5X8=40)**
ANSWER ANY FIVE QUESTIONS

11. A bag contains 6 white and 9 black balls. Four balls are drawn at random. Find the probability for the first draw to give 4 white and the second to give 4 black balls in the each of the following cases.

    (i) The balls are replaced before the second draw.

    (ii) The balls are not replaced before the second draw.
12. The joint pdf of two random variables X and Y is

    $$f(x,y) = f(x) = \begin{cases} \frac{1}{8}x(x-y), \ 0 < x < 2 \\ \qquad\qquad -x < y < x \\ 0 \qquad elsewhere \end{cases}$$

    Obtain (i) the marginal distribution of X and Y (ii) the conditional distribution of Y for $X = x$ given.

13. State and prove Tchebychev's inequality.
14. Determine the binomial distribution for which the mean is 4 and variance is 3. Also find its mode.
15. Obtain the m.g.f. of normal distribution $N(\mu, \sigma)$.
16. State and prove Boole's inequality for events.
17. State and prove additive property of Poisson random variable.

SECTION – C **(2X20=40)**
ANSWER ANY TWO QUESTIONS

18. a) State and prove Baye's theorem.

 b) The contents of urns I, II and III as follows:

 1 white, 2 red and 3 black balls,

 2 white4, 3 red and 1 black ball and

 3 white, 1 red and 2 black balls.

 One urn is chosen at random and two balls are drawn. They happens to be white and red. What is the probability that they come from urns I, II or III.

19. a) Prove that the m.g.f. of the sum of a number of independent random variables is equal to the product of their respective m.g.f s.

 b) If X and Y are independent Poisson variates much that $P(X = 1) = P(X = 2)$ and $P(Y = 2) = P(Y = 3)$. Find the variance of $X - 2Y$.

20. a) X is a normal variate with mean 30 and S.D. 5. Find the probabilities that (i) $26 \leq X \leq 40$, (ii) $X \geq 45$ and (iii) $|X - 30| > 5$.

 b) State and prove area property of a normal random variable.

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