STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2011–12 & thereafter)

SUBJECT CODE : 11MT/MC/AS54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2015
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE	**:**	**MAJOR – CORE**
PAPER	**:**	**ALGEBRAIC STRUCTURES**
TIME	**:**	**3 HOURS** — **MAX. MARKS : 100**

SECTION – A

Answer all questions: **(10 X 2 =20)**

1. Prove that the identity element of a group is unique.
2. Prove that the intersection of two subgroups of a group is a subgroup of the group.
3. Prove that any subgroup of an abelian group is normal.
4. If $\varphi : G \rightarrow G'$ is a homomorphism of groups, prove that $\varphi(e) = e'$, where e and e' are respectively the identity elements of G and G'.
5. What is the number of automorphisms of the group { 1, -1. i, -i} under multiplication.
6. Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$ as the product of disjoint cycles.
7. Prove that any field is an integral domain.
8. Define an integral domain and give an example of a ring which is not an integral domain.
9. Define a maximal ideal of a ring.
10. If U is an ideal of a ring R and if $1 \in U$, prove that $U = R$.

SECTION – B

Answer any five questions: **(5 X 8 = 40)**

11. State and prove the necessary and sufficient conditions for a subset H of a group G to be a subgroup of G.
12. If $\varphi: G \rightarrow G'$ is a homomorphism of groups, prove that the kernel of φ is a normal subgroup of G.
13. Find the set of automorphism of an infinite cyclic group.
14. Prove that a finite integral domain is a field.

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15. Let R be a commutative ring with unit element and M is an ideal of R. Prove that M is a maximal ideal of R if and only R/M is a field.
16. If G is a group and if H is a subgroup of H of index 2 in G, prove that H is a normal subgroup of G.
17. Prove that the set A_n of even permutations on *n* symbols is a normal subgroup of the symmetric group of S_n.

SECTION – C

Answer any two questions: **(2 x 20 = 40)**

18. (a) Let $G = J$ be the group of integers under usual addition and $H = 5J$ be the subgroup of G consisting of all integers which are multiples of 5. Write down the right cosets of H in G. Is the set of right cosets of H in G a group? Give reasons.

    (b) If G is a group of even order, prove that there exists an element $a \neq e$ in G such that $a^2 = e$.

    (c) State and prove Lagrange's theorem on finite groups. (5+5+10)

19. (a) State and prove Cayley's theorem.

    (b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field. (10 + 10)

20. (a) Prove that any integral domain can be imbedded in a field.

    (b) Give an example of division ring which is not a field. (15 + 5)

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