STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16)

SUBJECT CODE : 15MT/PC/RA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2015
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : REAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 2 = 10)**

ANSWER ALL QUESTIONS

1. Define accumulation point. What is the accumulation point of the set $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$?
2. Show that $\sum_{1}^{\infty}(-1)^{(n-1)}$ is $(C,1)$ summable.
3. State Weierstrass M-test.
4. Define linear function. If $\vec{f}$ is linear show that $\vec{f}'(\vec{c};\vec{x}) = \vec{f}(\vec{x})$.
5. Find the saddle points of the function $f(x,y) = (y-x^2)(y-2x^2)$.

SECTION – B **(5 X 6 = 30)**

ANSWER ANY FIVE QUESTIONS

6. Prove that every point of a non empty open set S belongs to one and only one component interval of S .
7. Prove that a set S in R^n is closed if and only if it contains all its adherent points.
8. Given $f(p,q) = \frac{pq}{p^2+q^2}$, find the double limit and the iterated limit if it exists.
9. Assume that $\{f_n\}$ converges uniformly to f on S . If each f_n is continuous at a point c of S the prove that the limit of the function f is also continuous at c .
10. Let S be an open connected subset of R^n . Let $\vec{f} : S \to R^n$ be differentiable at each point of S . If $\vec{f}'(\vec{c}) = 0$, $\forall \vec{c} \in S$ then prove that $\vec{f}$ is a constant on S .
11. State and prove Taylor's formula from R^n to R^1 .
12. Let A be an open subset of R^n and assume that $\vec{f} : A \to R^n$ is continuous and has finite partial derivative $D_j f_i$ on A . If $\vec{f}$ is one to one on A and if $J_{\vec{f}}(\vec{x}) \neq 0, \forall x \in A$, then prove that $\vec{f}(A)$ open.

SECTION – C **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

13. a) State and prove Cantor intersection theorem.

 b) Assume $A \subseteq R^n$ and F be an open covering of A. Then prove that there is a countable sub covering of F which also covers A.

(10+10)

14. a) State and Prove Merten's theorem.

 b) State and prove Cauchy condition for convergence of products.

(10+10)

15. a) Define pointwise convergence and uniform convergence of functions. Does pointwise convergence implies uniform convergence. Justify your answer.

 b) State and Prove Bernstein's theorem.

(8+12)

16. a) Assume that one of the partial derivatives $D_1\vec{f}, D_2\vec{f}, D_3\vec{f}.........D_n\vec{f}$ exists at $\vec{c}$ and that the remaining $(n-1)$ derivatives exist in some ball $B(\vec{c})$ and are continuous at $\vec{c}$, then prove that $\vec{f}$ is differentiable at $\vec{c}$.

 b) If both the partial derivatives $D_r\vec{f}$ and $D_k\vec{f}$ exist in an n - ball $B(\vec{c};\delta)$ and if both are differentiable at $\vec{c}$, then prove that $D_{k,r}\vec{f}(\vec{c}) = D_{r,k}\vec{f}(\vec{c})$.

(12+8)

17. State and prove Inverse function theorem.