STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16)

SUBJECT CODE : 15MT/PC/DE14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2015
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : DIFFERENTIAL EQUATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A (5 X 2 = 10)

ANSWER ALL QUESTIONS

1. Check whether the functions $x_1(t) = e^{i\alpha t}, x_2(t) = sin\ \alpha t, x_3(t) = cos\ \alpha t$ are linearly dependent on $-\infty < t < \infty$.
2. State Lipschitz condition.
3. Write the auxillary equations of Charpit's method.
4. Define boundary value problems.
5. Write down the Laplace's equation in two dimensional polar coordinates.

SECTION – B (5 X 6 = 30)

ANSWER ANY FIVE QUESTIONS

6. If $x_1(t)$ and $x_2(t)$ are linearly independent solutions of the equation $L(x) = 0$ on I, then prove that the Wronskian $W[\, x_1(t), x_2(t)]$ of $x_1(t)$ and $x_2(t)$ is never zero on I.
7. If $P_m(t)$ and $P_n(t)$ are Legendre polynomials, then prove that $\int_{-1}^{+1} P_m(t)\, P_n(t)\, \text{dt} = 0$ if $m \neq n$.
8. Solve the initial value problem $x' = x,\ x(0) = 1,\ t \geq 0$ by the method of successive approximation.
9. Solve the equation $px + qy + pq = 0$ by Charpits method.
10. Obtain the general solution of heat flow equation $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ by the method of separation of variables.
11. Obtain a solution of Laplace equation in rectangular Cartesian coordinates (x, y, z) by the method of separation of variables.
12. State and prove Gronwall inequality.

SECTION – C **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

13. a) Prove that $t^{\frac{1}{2}} J_{\frac{1}{2}}(t) = \frac{\sqrt{2}}{\Gamma(\frac{1}{2})} sint$ and that $t^{\frac{1}{2}} J_{-\frac{1}{2}}(t) = \frac{\sqrt{2}}{\Gamma(\frac{1}{2})} cost.$

    b) Solve the Legendre equation $(1-t^2)x'' - 2tx' + p(p+1)x = 0$, when p is a real number.

14. a) State and prove Picard's theorem for initial value problems.

    b) Solve the boundary value problem $x'' + \lambda x = 0,\ x(0) = 0, x'(1) = 0.$

15. a) Find the complete integral of $p^2x + q^2y - z = 0.$

    b) Derive one dimensional wave equation.

16. a) Obtain the solution of the two dimensional heat equation $\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y2} = \frac{1}{k}\frac{\partial\theta}{\partial t}$.

    b) Solve by the method of separation of variables: $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0.$

17. a) Solve the differential equation $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial\theta^2} = 0$ subject to the boundary conditions i) u is finite when r → 0 ii) $u = \sum C_n \cos n\theta$ when $r = a.$

    b) State and prove Dirichlet problem on harmonic functions.

⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄