STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16)

SUBJECT CODE : 15MT/PC/CM14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2015
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : CONTINUUM MECHANICS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 2 = 10)**

ANSWER ALL THE QUESTIONS

1. Define isotropic.
2. The Lagrangian description of a deformation is given by $x_1 = X_1 + X_3(e^2 - 1)$, $x_2 = X_2 + X_3(e^2 - e^{-2})$, $x_3 = X_3$. Determine the Eulerian equation describing the motion.
3. Define steady motion.
4. Show that the material form $d(\rho J)/dt = 0$ of the continuity equation and the spatial form $d\rho/dt + \rho V_{u,u} = 0$ are equivalent.
5. Define orthotropic.

SECTION – B **(5 X 6 = 30)**

ANSWER ANY FIVE QUESTIONS

6. Show that the deviator stress tensor is equivalent to the superposition of five simple shear states.
7. Obtain the three Lagrangian strain invariants.
8. A continuum under goes deformation $x_1 = X_1$, $x_2 = X_2 + AX_3$, $x_3 = X_3 + AX_2$ where A is a constant. Compute the deformation tensor G and use this to determine the Lagrangian finite strain Tensor L_G.
9. Show that the rate of deformation tensor is the material derivative of the Eulerian linear strain tensor.
10. A velocity field is given by $v_1 = 4x_3 - 3x_2$, $v_2 = 3x_1$, $v_3 = -4x_1$. Determine the acceleration components at $P(b, 0,0)$ and $Q(0,4b, 3b)$.
11. Discuss Angular Momentum principle.
12. Obtain the generalized Hook's law.

SECTION – C (3 X 20 = 60)

ANSWER ANY THREE QUESTIONS

13. The stress tensor values at a point P are given by the array

$$\varepsilon = \begin{pmatrix} 7 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{pmatrix}$$

(i) Determine the traction vector on the plane at P where unit normal is
$\hat{n} = \left({}^{2}/{}_{3}\right)\hat{e}_1 - \left({}^{2}/{}_{3}\right)\hat{e}_2 + \left({}^{1}/{}_{3}\right)\hat{e}_3$

(ii) The component perpendicular to the plane

(iii) The magnitude of $\overset{(\hat{n})}{t_i}$.

(iv) The angle between $\overset{(\hat{n})}{t_i}$ and $\hat{n}$.

14. Obtain Lagrangian finite strain tensor and Eulerian finite strain tensor. Also obtain Cauchy's and Green's deformation tensor.

15. A velocity field is described by $v_1 = x_1/(1+t)$, $v_2 = 2x_2/(1+t)$, $v_3 = 3x_3/(1+t)$. Determine the acceleration components for the motion and obtain the displacement relations $x_i = x_i(\vec{X}, t)$ are also from these determine the acceleration components in Lagrangian form for the motion.

16. In a two dimensional incompressible steady flow $v_1 = -Ax_2/r^2$ where $r^2 = {x_1}^2 + {x_2}^2$. Determine v_2 if $v_2 = 0$ at $x_1 = 0$ for all x_2. Show that the motion is irrotational and that the streamlines are circles.

17. a) Derive Hook's Law for an isotropic body in terms of Lame's constants λ and μ.

b) With usual notations obtain (C_{KM}) where x_1x_2 plane is one of elastic symmetry, under the transformation $x_1' = x_1,\ x_2' = x_2,\ x_3' = x_3$.