STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16)

SUBJECT CODE : 15MT/PC/MA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2015
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : MODERN ALGEBRA
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(5 x 2 = 10)**

1. Does there exist a non-abelian group of order 169? Justify your answer.
2. Define an Euclidean ring.
3. If p is a prime number, show that the polynomial $x^n - p$ is irreducible over the field of rational numbers.
4. Is $Q(e)$ a finite extension of Q, where Q is the field of rational numbers?
5. Is the symmetric group S_3 solvable? Give reasons for your answer.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 x 6 = 30)**

6. Prove that any two p-Sylow subgroups of a finite group G are conjugates.
7. Let R be an integral domain. Prove that any two elements a and b in R have greatest common divisor d and it is of the form $d = \lambda a + \mu b$, for some λ and μ in R.
8. If $f(x)$ and $g(x)$ are primitive polynomials, prove that $f(x)g(x)$ is also a primitive polynomial.
9. If $a, b \in K$ are algebraic over F of degrees m and n respectively, and if m and n are relatively prime, prove that $F(a, b)$ is of degree mn over F.
10. Prove that a group G is solvable if and only if $G^{(k)} = (e)$ for some k.
11. Let G be a group and suppose that G is the internal direct product of $N_1, N_2, \dots, N_n$. Let $T = N_1 x\, N_2 x \,\dots\, x\, N_n$. Prove that the groups G and T are isomorphic.
12. Prove that a polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non-trivial common factor.

SECTION – C

ANSWER ANY THREE QUESTIONS: (3 x 20 = 60)

13. Prove that every finite abelian group is the direct product of cyclic groups.
14. (a) Prove that an ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R if and only if (a_0) is a prime element of R.

    (b) Prove that the ring if Gaussian integers is a Euclidean ring.
15. (a) State and prove the Eisenstein criterion about the irreducibility of a polynomial with integer coefficients.

    (b) State and prove the division algorithm of polynomials over a field F.
16. Prove that the number e is transcendental.
17. Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.