STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2011 – 12 & thereafter)

SUBJECT CODE: 11MT/PC/GT34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2015
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : GRAPH THEORY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 2 = 10)**

ANSWER ALL THE QUESTIONS

1. Define graph isomorphism.
2. Define covering and minimum covering of a graph.
3. Prove that every critical graph is a block.
4. State Kuratowski's theorem.
5. Define diameter of a graph.

SECTION – B **(5 X 6 = 30)**

ANSWER ANY FIVE QUESTIONS

6. If G is a tree, Prove that $\varepsilon = v - 1$.
7. Prove that a matching M in G is a maximum matching if and only if G contains no M – augmenting path.
8. If $\delta > 0$, Prove that $\alpha' + \beta' = v$.
9. Prove that a nonempty connected graph is eulerian if and only if it has no vertices of odd degree.
10. If G is a simple graph, Prove that $\pi_k(G) = \pi_k(G - e) - \pi_k(G.e)$ for any edge e of G.
11. Prove that K_5 is non-planar.
12. State the properties of circulant digraphs $G(n; s_1, s_2, \ldots, s_k)$.

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SECTION – C (3 X 20 = 60)

ANSWER ANY THREE QUESTIONS

13. a) Prove that a graph is bipartite if and only if it contains no odd cycle.
    b) Prove that an edge *e* of *G* is a cut edge of *G* if and only if *e* is contained in no cycle of *G*. (10 +10)

14. a) With usual notations prove that $\kappa \leq \kappa' \leq \delta$.
    b) If *G* is a bipartite graph with bipartition (*X*, *Y*), Prove that *G* contains a matching that saturates every vertex in *X* if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$.
    (10 + 10)

15. a) State and prove Brooks' theorem.
    b) If *G* is a simple graph with $v \geq 3$ and $\delta \geq v/2$, Prove that *G* is Hamiltonian.
    (12 + 8)

16. a) State and prove Euler's formula for connected plane graph.
    b) State and prove the five colour theorem. (8 + 12)

17. a) Explain any five basic principles of network design.
    b) State the properties of Kautz network $K(d, n)$.
    c) Compare de Bruijn network and Hypercube network. (10 + 5 + 5)