**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**
**(For candidates admitted during the academic year 2011 – 12 & thereafter)**

**SUBJECT CODE: 11MT/PC/CA34**

**M. Sc. DEGREE EXAMINATION, NOVEMBER 2015**
**BRANCH I - MATHEMATICS**
**THIRD SEMESTER**

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|---|---|---|---|
| **COURSE** | **: CORE** | | |
| **PAPER** | **: COMPLEX ANALYSIS** | | |
| **TIME** | **: 3 HOURS** | **MAX. MARKS :** | **100** |

## SECTION – A

**ANSWER ALL QUESTIONS:** **(5 × 2 = 10)**

1. State the general statement of Cauchy Theorem explaining a cycle.
2. Prove that any harmonic function which depends only on $r$ is of the form $a \log r + b$ where $a$ and $b$ are constants.
3. Prove that $\zeta(s)\Gamma(s) = \int_0^\infty \frac{x^{s-1}}{e^x - 1} dx$.
4. State True of False and justify your answer with a suitable example : " The derivatives of a normal family form a normal family ".
5. When do we say $\phi(t)$ determines an analytic arc.

## SECTION – B

**ANSWER ANY FIVE QUESTIONS:** **(5 × 6 = 30)**

6. Prove that a region $\Omega$ is simply connected if and only if $n(\gamma, a) = 0$ for all cycles $\gamma$ in $\Omega$ and all points a which do not belong to $\Omega$.
7. Prove that a non constant harmonic function has neither a maximum nor a minimum in its region of definition.
8. Obtain a product representation of $Sin\pi z$.
9. Deduce Legendre's duplication formula using the relation $\xi(s) = \xi(1-s)$ where $\xi(s) = \frac{1}{2} s(1-s)\Pi^{\frac{-s}{2}} \Gamma\left(\frac{s}{2}\right)\zeta(s)$.
10. Prove that a family $\Im$ is normal if and only if its closure $\overline{\Im}$ with respect to the distance function $\rho(f, g) = \sum_{k=1}^{\infty} \delta_k(f, g) 2^{-k}$ is compact.
11. Show that convergence with respect to $\rho$ is equivalent to uniform convergence on all compact sets.
12. Prove that the functions $z = F(w)$ which map $|w| < 1$ conformally onto polygons with angles $\alpha_k \pi (k = 1,2,........ n)$ are of the form $F(w) = C \int_0^w \prod_{k=1}^{n} (w - w_k)^{-\beta_k} dw + C'$, where $\beta_k = 1 - \alpha_k, w_k$ are points on the unit circle and $C$ and $C'$ are complex constants.

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## SECTION – C

**ANSWER ANY THREE QUESTIONS:** **$(3 \times 20 = 60)$**

13. (a) State and prove Cauchy's Theorem for a Rectangle.

(b) If the piecewise differentiable closed curve $\gamma$ does not pass through the point a then, prove that the value of the integral $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$.

14. (a) Suppose that $u(z)$ is harmonic for |z|<R, continuous for $|z| \leq R$, prove that $u(a) = \frac{1}{2\pi} \int_{|z|=R} \frac{R^2 - |a|^2}{|z-a|^2} u(z) d\theta$ for all $|a| < R$.

(b) State and prove Jensen's formula.

15. (a) For $\sigma = Re(s) > 1$, prove that $\zeta(s) = -\frac{\Gamma(1-s)}{2\pi i} \int_C \frac{(-z)^{s-1}}{e^z - 1} dz$, where $(-z)^{s-1}$ is defined on the complement of the positive real axis as $e^{(s-1)\log(-z)}$ with $-\pi < \operatorname{Im}\log(-z) < \pi$ .

(b) Prove that $\zeta(s) = 2^s \Pi^{s-1} Sin \frac{\pi s}{2} \Gamma(1-s) \zeta(1-s)$.

16. State and prove Arzela Ascoli theorem.

17. State and prove Riemann Mapping theorem.

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