

B. Sc. DEGREE EXAMINATION, NOVEMBER 2008
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – OPTIONAL
PAPER : INTRODUCTION TO GRAPH THEORY
TIME : 3 HOURS
MAX. MARKS : 100

SECTION – A (20 X 1 = 20)

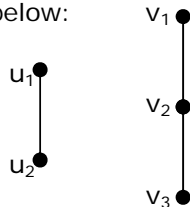
1 State whether the following statements are true or false:

1. Any graph with p points is isomorphic to a subgraph of K_p .
2. There are exactly twelve non-isomorphic simple graph on 4 vertices.
3. If $\delta \geq 2$ then G contains a cycle.
4. A regular graph of degree 0 is totally disconnected.
5. If G contains no odd cycle, then G is bipartite.
6. In a tree, any two vertices are connected by exactly two paths.
7. If a graph has a cutpoint, then it has a bridge.
8. The complement of a connected graph is connected.
9. Every tree with exactly 2 vertices of degree 1 is a path.
10. Every tree is a bipartite graph.
11. Every Eulerian digraph is strongly connected.
12. Every point of a digraph lies in exactly two strong components.
13. Every point of a digraph lies in exactly one weak component.
14. Every planar graph is 4-colourable.
15. If G is regular, then $G - v$ is regular.
16. Any $u - v$ walk contains a $u - v$ path.
17. The chromatic number of a tree with atleast two points is three.
18. The chromatic number of any cycle is 2.
19. The Petersen graph is Eulerian.
20. The Petersen graph is not Hamiltonian.

SECTION – B

ANSWER ANY FIVE OF THE FOLLOWING (5x8=40)

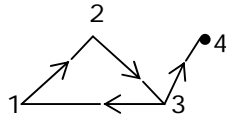
2. Prove that in any graph $\delta \leq \frac{2q}{p} \leq \Delta$.
3. A graph G with atleast two points is bipartite iff all its cycles are of even length.
4. If G is a bipartite graph with odd number of vertices then prove that G is Hamiltonian.
5. Determine $G_1 \cup G_2$, $G_1 + G_2$, $G_1 \times G_2$ and $G_1[G_2]$ where G_1 and G_2 are given below:



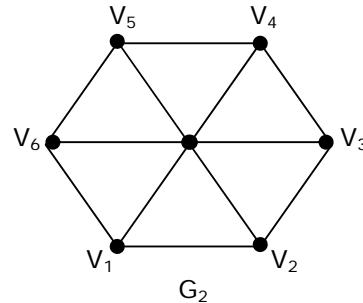
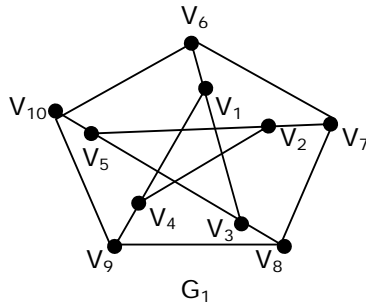
6. If two digraphs are isomorphic then show that their corresponding points have the same degree pair.
7. If G is a (p, q) connected graph, then show that $q \geq p - 1$.
8. Draw a graph which is neither Eulerian nor Hamiltonian and a graph which is Eulerian but not Hamiltonian.

SECTION – C
ANSWER ANY TWO OF THE FOLLOWING **(2X20=40)**

9. a) Let G be a (p, q) graph. Prove that (i) if any two points of G are joined by a unique path, then G is connected and $p = q + 1$ (ii) if G is a cyclic and $p = q + 1$ then G is a tree.
 b) Prove that every planar graph is 5-colourable. (10+10)
10. a) Find the adjacency matrix and the incidence matrix of the digraph given below:



- b) Prove that a graph G is connected iff for any partition of V into subsets V_1 and V_2 there is a line of G joining a point of V_1 to a point of V_2 .
- c) Prove that if G is a graph with $p \geq 3$ vertices and $\delta \geq \frac{p}{2}$, then G is Hamiltonian.
11. a) Prove that if G is a plane (p, q) graph with r faces and k components then $p - q + r = k + 1$.
 b) Prove that k_5 and $k_{3,3}$ are not planar.
 c) Find the chromatic number of the following graphs.



(10+5+5)

