STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2004 – 05 & thereafter)

SUBJECT CODE : MT/MC/AS54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2008
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE	:	**MAJOR – CORE**
PAPER	:	**ALGEBRAIC STRUCTURES**
TIME	:	**3 HOURS**

MAX. MARKS : 100

SECTION – A **(10 X 2 = 20)**

ANSWER ALL THE QUESTIONS

1. Define an abelian group. Give an example.
2. Prove that for all $a \in G$ (G is a group) $\left(a^{-1}\right)^{-1} = a$.
3. State left and right Cancellation Laws.
4. Prove that intersection of two subgroups of a group is again a subgroup.
5. Define a commutative ring.
6. If $\phi : R \to R'$ is a ring homomorphism, then prove that $\phi(0) = 0$.
7. If $\phi : G \to G'$ be a group homomorphism, then prove that $Ker\phi$ is a normal subgroup.
8. Define maximal ideal of a ring.
9. Define order of an element in a group G.
10. If R is a commutative ring and $a \in R$. Prove that $aR = \{ar / r \in R\}$ is a two-sided ideal of R.

SECTION – B **(5X8=40)**

ANSWER ANY FIVE QUESTIONS

11. State and prove Lagrange's Theorem.
12. State and Prove Necessary and Sufficient Condition for a non-empty subset of a group G to become a subgroup of G.
13. If G is a finite group and $a \in G$, then prove that $o(a)$ divides $o(G)$.
14. Prove that N is a normal subgroup if and only if $gNg^{-1} = N$, for every $g \in G$.
15. Prove that every field is an integral domain.
16. Prove that a finite integral domain is a field.
17. If U is an ideal of R and $1 \in U$, prove that $U = R$.

SECTION – C **(2X20=40)**

ANSWER ANY TWO QUESTIONS

18. a) If H and K are finite subgroups of G of order $o(H)$ and $o(K)$ respectively, then prove that $o(HK) = \frac{o(H) \cdot o(K)}{o(H \cap K)}$ (10 marks)

    b) Define center of a group G. Prove that the center of a group G is a normal subgroup of G. (10 marks)

19. a) If $\phi : G \to \overline{G}$ is a group homomorphism of G onto $\overline{G}$ with kernel K. Then prove that $\frac{G}{K} \cong \overline{G}$. (10 marks)

    b) Prove that the product of two odd permutation is an even permutation. (5 marks)

    c) Prove that only ideals of a field F are $\{0\}$ and F itself. (5 marks)

20. Prove that every integral domain can be imbedded in a field.

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