

STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086  
(For candidates admitted from the academic year 2011–12 & thereafter)

SUBJECT CODE : 11MT/ME/FM63

B. Sc. DEGREE EXAMINATION, APRIL 2015  
BRANCH I – MATHEMATICS  
SIXTH SEMESTER

COURSE : MAJOR ELECTIVE  
PAPER : FINANCIAL MATHEMATICS  
TIME : 3 HOURS

MAX. MARKS : 100

SECTION-A

ANSWER ALL QUESTIONS:

10 X 2 = 20

1. Define Brownian motion.
2. State the mean and variance of Geometric Brownian motion.
3. Define effective interest rate.
4. Define continuously varying interest rates.
5. State the law of one price.
6. Argue that one should never exercise an American style call option before its expiration time  $t$ .
7. With usual notation, prove that  $\frac{\partial C}{\partial K} = e^{-rt} \varphi(\omega - \sigma\sqrt{t})$ .
8. Prove that  $C(s, t, K, \sigma, r)$  is an increasing function in  $s$ .
9. Define VAR and CVAR.
10. Write down the limitations of Arbitrage pricing.

SECTION-B

ANSWER ANY FIVE QUESTIONS:

5 X 8 = 40

11. Define Geometric Brownian motion and prove that it is a limit of simpler models.
12. Find the yield curve and the present value function if  $r(s) = \frac{r_1}{1+s} + \frac{s r_2}{1+s}$ .
13. State and prove Arbitrage Theorem.
14. Explain delta hedging arbitrage strategy.
15. An investor with capital  $x$  can invest any amount between 0 and  $x$ ; if  $y$  is invested then  $y$  is either won or lost, with respective probabilities  $p$  and  $1 - p$ . If  $p > 1/2$ , how much should be invested by an investor having a log utility function?

16. State and prove put-call option parity formula.
17. Define rate of return of an investment and suppose that you are to receive payments (in thousands of dollars) at the end of each of the next five years. Which of the following three payment sequences is preferable if the interest rate compounded annually at 10%?  
 A : 12, 14, 16, 18, 20      B : 16, 16, 15, 15, 15      C : 20, 16, 14, 12, 10.

### SECTION-C

ANSWER ANY TWO QUESTIONS:

2 X20 = 40

18. a) An individual who plans to retire in 20 years has decided to put an amount  $A$  in the bank at the beginning of each of the next 240 months, after which she will withdraw \$1000 at the beginning of each of the following 360 months. Assuming a nominal yearly interest rate of 6% compounded monthly, how large does  $A$  need to be? (10)
- (b) Explain the Law of One Price by an example. (10)
19. Derive the Black-Scholes option cost formula,  $C = s\phi(\omega) - Ke^{-rt}\phi(\omega - \sigma\sqrt{t})$ .
20. (a) Suppose you are investing your fortune of 100 in two securities whose rates of return have the following expected values and standard deviations:  $r_1 = 0.15$ ,  $v_1 = 0.2$ ,  $r_2 = 0.18$ ,  $v_2 = 0.25$ . If the correlation between the rates of return is  $\rho = -0.4$ , find the optimal portfolio when employing the utility function  $U(x) = 1 - e^{-0.005x}$ . (10)
- (b) Explain mean variance analysis of risk-neutral-priced call options. (10)

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