STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI - 600086
(For candidates admitted during the academic year 2011-12\& thereafter)

## B.Sc/ B.A/ B.Com Degree End Semester Examination - April 2015

SUBJECT CODE: 11MT/GE/DM44
COURSE : GENERAL ELECTIVE
MAXIMUM MARKS : 100
PAPER : DISCRETE MATHEMATICS
TIME : 3 HOURS
SECTION - A (10 x2 =20)

## Answer ALL TheQuestions:

1. Verify whether $(P \vee Q) \vee 7(P \wedge Q)$ is a contradiction or tautology.
2. Determine the truth value of the following statement:

$$
\mathrm{U}=\{1,2,3,4,5\} ; \exists x \forall y, 5 x-y \leq 15
$$

3. Define a partially ordered set with an example.
4. What is Hasse diagram?
5. State the Duality and Idempotent laws in a lattice.
6. Let $N=\{1,2,3, \ldots\}$ be ordered by divisibility. State whether each of the following subsets of N are linearly ordered:
i) $\{2,4,18\}$; ii) $\{3,21,9\}$;
7. Write the dual of each Boolean expression $(a * 1)+\left(a^{\prime}+0\right)=a+b$.
8. Write the output sequence $Y$ for a NOT gate with input $A$ where $A$ is given as follows:
i) 00111110; ii) 11100111;
9. Explain regular expressions and regular Languages.
10. What are Godel Numbers?

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\text { SECTION }-B \quad(5 \times 8=40)
$$

## Answer Any Five Questions:

11. a) State De Morgan's theorem.
b) Draw the truth table of $((p \rightarrow q) \wedge p) \rightarrow \neg q$.
12. Let $S=\{a, b, c, d, e\}$ be ordered as in the following Hasse Diagram.

a) Find all minimal and maximal elements of $S$.
b) Does $S$ have a first element or a last element?
c) Is $d$ and $a$ comparable?
d) Is $c<b$ ?
13. a) Suppose the English Alphabet $A=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \ldots\}$ is given the usual alphabetical order $\operatorname{and} A^{2}=A x A$ is given the product order. Insert the correct symbol $<,>$ or $\prec, \succ$ or $\|$ :
i) $y d$ $u s$; ii) $g y$ $\qquad$ $g t$; iii) $c a$ $\qquad$ $x c ;$
b) Let $C=\{1,2, \ldots, 16\}$ be ordered by divisibility. Draw the Hasse diagram for $C$ and check whether it is a Poset.
14. a) What are Isomorphic Lattices? Give an example.
b) Check whether the given Hasse diagram is Lattice.
c) If so determine whether it is distributive and bounded?
d) Identify the atoms in it.

15. Consider the Boolean Algebra $D_{210}$.
a) List its elements and draw its diagram.
b) Find the set of atoms.
c) Find a subalgebra with eight elements.
d) Is $X=\{1,2,7,70\}$ a sublattice of $D_{210}$ ?
16. a) Consider the words $u=a^{2} b^{2} a^{3}$ and $v=b a b^{3}$. Find $u v$ and $v^{2}$.
b) Explain Finite state Automata.
17. Let $L$ be the set of all words in $a$ and $b$ with an even number of $a$ 's. Find a grammar $G$ which will generate $L$.

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\text { SECTION }-\mathrm{C} \quad(2 \times 20=40)
$$

## Answer Any Two Questions:

18. a) Determine the validity of the following argument: $(p \rightarrow q), \neg q \mapsto \neg p$.
b) Define the two quantifiers and Negate each of the following statements:
i) $(\exists z \in A)(z-3<8)$;
ii) $(\forall x \in A)\left(27-x^{2}<9\right)$
iii) $\exists x \forall y, q(x, y)$;
iv) $\forall x \forall y \exists z, \neg p(x, y, z)$
19. a) Check whether the set $Z$ of integers and $Q$ set of rational numbers, with the usual order $\leq$, is linearly ordered and well ordered.
b) Let $T=\{2,3,6,12,24,48\} . R$ on $T=\{(x, y) \in \mathrm{R}, x$ divides $y\}$
i) Construct Hasse diagram.
ii) Find maximal and minimal element.
iii) Give chains and antichains.
iv)Find the maximal length of chains.
v) Is this poset a lattice?
20. a) Explain the logical gates with an example and mention few of their applications.
b) If a Finite Automaton $M$ is given by $M=\left(S, A, f, Y, s_{0}\right)$, where
$S=\left\{s_{0}, s_{1}, s_{2}, s_{3}\right\}, A=\{0,1\}, Y=\left\{\mathrm{s}_{0}, \mathrm{~s}_{2}\right\}$ and $f$ is given by the following table:

|  | 0 | 1 |
| :---: | :---: | :---: |
| $\mathrm{~S}_{0}$ | $\mathrm{~S}_{0}$ | $\mathrm{~S}_{1}$ |
| $\mathrm{~S}_{1}$ | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{2}$ |
| $\mathrm{~S}_{2}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ |
| $\mathrm{~S}_{3}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{0}$ |

Draw the state diagram and find the language of $M$.

