

STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI – 600 086

(For candidates admitted during the academic year 2011 – 12& thereafter)

B.Sc/ B.A/ B.Com Degree End Semester Examination - April 2015

SUBJECT CODE: 11MT/GE/BM24

BRANCH I – MATHEMATICS

COURSE : GENERAL ELECTIVE

PAPER : BASIC MATHEMATICS

TIME : 3 HOURS

MAXIMUM MARKS : 100

SECTION – A (10 x 2 = 20)

Answer ALL The Questions:

1. Define Hermitian and skew-Hermitian matrices with an example.
2. Find the determinant of the following matrix:  $\begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$
3. Give the formula for sum and product of all the roots of an equation:  $ax^3+bx^2+cx+d=0$ .
4. Find the equation one of whose root is  $\sqrt{7} - 5i$ .
5. State Cayley Hamilton's theorem.
6. Find the inverse of the following matrix:  $\begin{pmatrix} -1 & 0 \\ 5 & -2 \end{pmatrix}$
7. Write the derivative of  $\tan 7x$  with respect to  $x$ .
8. Differentiate  $\cos^{-1} \sqrt{3x}$  with respect to  $x$ .
9. Integrate  $\sqrt{1 + \sin 2x}$  with respect to  $x$ .
10. If  $x = a \cos \theta$  and  $y = a \sin \theta$  find  $\frac{dy}{dx}$ .

SECTION – B (5 x 8 = 40)

Answer Any Five Questions:

11. Verify whether  $(AB)^T = B^T \cdot A^T$ , if  $A = \begin{pmatrix} 0 & 6 & -5 \\ 4 & 3 & 1 \end{pmatrix}$  and  $B = \begin{pmatrix} 4 & -7 \\ -2 & 5 \\ 3 & 0 \end{pmatrix}$ .

12. Find the rank of the matrix  $A = \begin{bmatrix} 1 & 1 & 1 & -4 \\ -1 & 0 & 6 & 8 \\ 3 & 4 & 10 & -12 \end{bmatrix}$ .

13. Solve the equation  $x^4 - 11x^2 + 2x + 12 = 0$  given that  $\sqrt{5} - 1$  is a root.

14. Solve the equation  $2x^3 - x^2 - 22x - 24 = 0$  given that two of its roots are in the ratio 3 : 4.

15. Verify Cayley–Hamilton theorem for the matrix  $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{bmatrix}$  and hence find  $A^{-1}$ .

16. Differentiate with respect to x:

(i)  $\sin^{-1}\left(\frac{3x}{4}\right)$       (ii)  $x \tan^{-1} x$

17. Find  $\frac{dy}{dx}$ , if  $x = a\left(\sin \theta + \log \tan \frac{\theta}{2}\right)$  and  $y = a \cos \theta$ .

**SECTION – C      (2 x 20 =40)**

**Answer Any Two Questions:**

18. Find the eigen values and eigen vectors of the matrix  $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ .

19. a) If one root of the equation  $x^3 + ax + b = 0$  is twice the difference of the other two, prove that one root is  $\frac{13b}{3a}$ .

b) If two roots of the equation  $x^4 - 6x^3 + px^2 + qx + 25 = 0$  are of the form  $\alpha + i\beta$  and  $\beta + i\alpha$ , find all the roots and also the values of  $p$  and  $q$ . (10+10)

20.a) If  $x^y = e^{x-y}$ , prove that  $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$ .

b) Evaluate  $\int \frac{x^2}{(a+bx)^3} dx$ . (10+10)