STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/PE/FT24

M. Sc. DEGREE EXAMINATION, APRIL 2015
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : ELECTIVE
PAPER : FUZZY SET THEORY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION –A

Answer all the questions: **5×2=10**

1. Define the scalar cardinality of a fuzzy set A defined on a finite set X.
2. Define the domain and range of a binary fuzzy relation.
3. Define Yager class of fuzzy compliments.
4. Define a fuzzy number.
5. Define fuzzy negation.

SECTION –B

Answer any five questions: **5×6=30**

6. Let A, B be fuzzy subsets of . Then prove that ${}^{\alpha}A \cap {}^{\alpha}B = {}^{\alpha}(A \cap B)$, where $\alpha \in [0,1]$.
7. Let $f: X \to Y$ be a crisp function. Then for any fuzzy subset A of X and for $\alpha \in [0,1]$, prove that ${}^{\alpha}[f(A)] \supseteq f({}^{\alpha}A)$. Is the reverse side inclusion true? Justify your answer.
8. Prove that every fuzzy complement has at most one equilibrium.
9. Determine which fuzzy sets defined by the following membership functions are fuzzy numbers?

   (i) $A(x) = \begin{cases} \sin x & for \quad 0 \le x \le \pi \\ 0 & otherwise \end{cases}$ (ii) $B(x) = \begin{cases} \min(1, x) & for \ x \ge 0 \\ 0 & for \ x \le 0. \end{cases}$
10. Define fuzzy extension of the classical AND and OR operator.
11. Order the following fuzzy sets defined by the following membership grade function (assuming $x \geq 0$) by the inclusion (subset) relation.

    $\underset{\sim}{A}(x) = \dfrac{1}{(1+10x)}, \quad \underset{\sim}{B}(x) = \left(\dfrac{1}{1+10x}\right)^{1/2}, \quad \underset{\sim}{C}(x) = \left(\dfrac{1}{1+10x}\right)^{2}$
12. . Prove that a fuzzy set A on $\mathcal{R}$ is convex if and only if

    $A(\lambda x_1 + (1-\lambda)x_2) \ge \min[A(x_1), A(x_2)], \forall x_1, x_2 \in R, \lambda \in [0,1].$

SECTION –C

Answer any three questions: **3×20=60**

13. Let $A_i \in \mathcal{F}(X)$ for all $i \in I$ (index set). Then prove that

(i) $\bigcup_{i\in I} {}^{\alpha}A_i \subseteq {}^{\alpha}\left(\bigcup_{i\in I} A_i\right)$.

(ii) $\bigcap_{i\in I} {}^{\alpha}A_i \subseteq {}^{\alpha}\left(\bigcap_{i\in I} A_i\right)$.

(iii) Is the reverse-side inclusion of (ii) true? Justify your answer.

(iv) $\bigcup_{i\in I} {}^{\alpha+}A_i = {}^{\alpha+}\left(\bigcup_{i\in I} A_i\right)$

(v) $\bigcap_{i\in I} {}^{\alpha+}A_i \subseteq {}^{\alpha+}\left(\bigcap_{i\in I} A_i\right)$.

14. Let $f : X \to Y$ be a crisp function. Then for any $A_i \in \mathcal{F}(X)$ and $B_i \in \mathcal{F}(X)$, prove the following properties obtained by the extension principle.

(i) $f(A) = \phi$ *iff* $A = \phi$.

(ii) If $A_1 \subseteq A_2$ *then* $f(A_1) \subseteq f(A_2)$

(iii) $f(\bigcup_{i\in I} A_i) = \left(\bigcup_{i\in I} f(A_i)\right)$.

(iv) $f(\bigcap_{i\in I} A_i) \subseteq {}^{\alpha}\left(\bigcap_{i\in I} f(A_i)\right)$.

15. Given a t-*conorm* u and an involutive fuzzy complement c , prove that the binary operation $i$on [0 , 1] defined by $i(a,b) = c(u(c(a), c(b)))$ for all $a, b \in [0,1]$ is a *t-norm* such that $\langle i, u, c \rangle$.

16. State and prove the necessary and sufficient conditions for a fuzzy subset of $\mathcal{R}$ to be a fuzzy number.

17. Explain any one application of fuzzy set theory in the field of medicine in detail.