STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2008-09)

SUBJECT CODE : MT/MC/SF44

B. Sc. DEGREE EXAMINATION, APRIL 2010
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE	:	MAJOR CORE
PAPER	:	SEQUENCES AND SERIES, FOURIER SERIES
TIME	:	3 HOURS MAX. MARKS : 100

SECTION – A

ANSWER ALL THE QUESTIONS: (10X2=20)

1. Define characteristic function of a set.
2. Define a countable set.
3. Define monotone sequence.
4. Define cauchy sequence.
5. If $\sum_{n=1}^{\infty} a_n$ is a convergent series, then prove that $\lim_{n\to\infty} a_n = 0$.
6. Give an example of a function which converges pointwise to f on $[0,1]$.
7. State the fundamental theorem on alternating series.
8. State comparison test.
9. Define absolute convergence of a series.
10. Define even and odd function.

SECTION – B

ANSWER ANY FIVE QUESTIONS: (5X8=40)

11. Prove that countable union of countable sets in countable.
12. Prove that if $\{s_n\}_{n=1}^{\infty}$ is a sequence of nonnegative numbers and if $\lim_{n\to\infty} S_n = L$ then $L \geq 0$.
13. If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} S_n = L$ and $\lim_{n\to\infty} t_n = M$ then prove that $\lim_{n\to\infty} S_n t_n = LM$.
14. State and prove the nested interval theorem.
15. Prove that if $\sum_{n=1}^{\infty} a_n$ converges absolutely then $\sum_{n=1}^{\infty} a_n$ converges.
16. State and prove Pringsheim's theorem.
17. Find a Sine series for $f(x) = c$ in the range 0 to π.

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2X20=40)**

18. a) Prove that $\left\{\frac{2n}{n+4n^{1/2}}\right\}_{n=1}^{\infty}$ converges to 2.

    b) Prove that $\left\{\left(1+{}^{1}/_{n}\right)^{n}\right\}_{n=1}^{\infty}$ is convergent.

19. a) State and prove the ratio rest.

    b) Discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ and prove that $\sum_{n=4}^{\infty}\frac{1}{(nlogn)}$ diverges.

20. Show that $x^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^n\cos nx}{n^2}$ in the interval $(-\pi\le x\le\pi)$.

    Deduce that (i) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\cdots=\frac{\pi^2}{12}$

    (ii) $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6}$

    (iii) $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}-\cdots=\frac{\pi^2}{8}$