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Application of Fuzzy Membership Matrix in Medical Diagnosis and Decision Making

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Abstract

The field of medicine and decision making are the most fruitful and interesting area of applications of fuzzy set theory. In real life situations, the imprecise nature of medical documentation and uncertain information gathered for decision making requires the use of "fuzzy". In this paper, procedures are presented for medical diagnosis and for fuzzy decision model. Examples are illustrated to verify the proposed approach.

Mathematics Subject Classification: 03B52, 92C50, 91B06

Keywords: Medical diagnosis, Decision making, Triangular fuzzy number matrices, New membership function, Fuzzy model, Network, Decision maker

1 INTRODUCTION

In recent years fuzzy set theory and fuzzy logic are highly suitable and applicable for developing knowledge based system in medicine for the tasks of medical findings. There are variety of models involving fuzzy matrices to deal with different complicated aspects of medical diagnosis. Sanchez formulated the diagnostic models involving fuzzy matrices representing the medical knowledge between symptoms and diseases [9, 10]. Esogbue and Elder [5] utilized fuzzy cluster analysis to model medical diagnostic. Meenakshi and Kaliraja

[7] have extended Sanchez's approach for medical diagnosis using the representation of a interval valued fuzzy matrix. They have also introduced the arithmetic mean matrix of an interval valued fuzzy matrix and directly applied Sanchez's method of medical diagnosis on it. Baruah [1, 2] used the definition of complement of a fuzzy soft set proposed by Neog and Sut [8] and put forward a matrix representation of fuzzy soft set and extended Sanchez's approach for medical diagnosis. Edward Samuel and Balamurugan [4] studied Sanchez's approach for medical diagnosis using Intuitionistic fuzzy set.

Fuzzy set theory also plays a vital role in the field of Decision Making. Decision Making is a most important scientific, social and economic endeavour. In classical crisp decision making theories, decisions are made under conditions of certainty but in real life situations this is not possible which gives rise to fuzzy decision making theories. For decision making in fuzzy environment one may refer Bellman and Zadeh [3]. Most probably the fuzzy decision model in which overall ranking or ordering of different fuzzy sets are determined by using comparison matrix, introduced and developed by Shimura [11].

The paper is organized as follows: In section 2, basic definitions of fuzzy set theory have been reviewed. In section 3, a novel approach is presented for medical diagnosis which is also an extension of Sanchez's approach with modified procedure using triangular fuzzy number matrices and its new membership function. In section 4, a procedure is proposed for fuzzy decision model using new relativity function and comparison matrix. In both the sections illustrative example is included to demonstrate the proposed approach. Section 5, concludes the paper.

2 PRE-REQUISITES

Definition 2.1 Triangular fuzzy number
Triangular fuzzy number is denoted as

$$A = (a_1, a_2, a_3), a_1, a_2, a_3 \in \Re, a_1 < a_2 < a_3.$$

Definition 2.2 Triangular fuzzy number matrix
Triangular fuzzy number matrix of order $m \times n$ is defined as $A = (a_{ij})_{m\times n}$ where $a_{ij} = (a_{ijL}, a_{ijM}, a_{ijU})$ is the ij^{th} element of A. a_{ijL}, a_{ijU} are the left and right spreads of a_{ij} respectively and a_{ijM} is the mean value.

Definition 2.3 Addition and Subtraction Operation on triangular fuzzy number matrix
Let $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ be two triangular fuzzy number matrices of same order. Then
(i) Addition Operation

$A(+)B = (a_{ij} + b_{ij})_{n\times n}$ *where*
$a_{ij} + b_{ij} = (a_{ijL} + b_{ijL}, a_{ijM} + b_{ijM}, a_{ijU} + b_{ijU})$ *is the* ij^{th} *element of* $A(+)B$
(ii) Subtraction Operation
$A(-)B = (a_{ij} - b_{ij})_{n\times n}$ *where*
$a_{ij} - b_{ij} = (a_{ijL} - b_{ijU}, a_{ijM} - b_{ijM}, a_{ijU} - b_{ijL})$ *is the* ij^{th} *element of* $A(-)B$
The same condition holds for triangular fuzzy membership number.

Definition 2.4 Multiplication Operation on Triangular fuzzy number matrix
Let $A = (a_{ij})_{m\times p}$ *and* $B = (b_{ij})_{p\times n}$ *be two triangular fuzzy number matrices. Then the Multiplication Operation:*
$A(\cdot)B = (c_{ij})_{m\times n}$, *where*
$$(c_{ij}) = \sum_{k=1}^{p} a_{ik} \cdot b_{kj} \text{ for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n.$$

Definition 2.5 Max-Min Composition on fuzzy membership value matrices
Let F_{mn} *denote the set of all* $m\times n$ *matrices over* F. *Elements of* F_{mn} *are called as fuzzy membership value matrices. For* $A = (a_{ij}) \in F_{mp}$ *and* $B = (b_{ij}) \in F_{pn}$ *the max-min product* $A(\cdot)B = (\sup_k[\{\inf\{a_{ik}, b_{kj}\}\}]) \in F_{mn}$.

Definition 2.6 Maximum Operation on triangular fuzzy number
Let $A = (a_{ij})_{n\times n}$ *where* $a_{ij} = (a_{ijL}, a_{ijM}, a_{ijU})$ *and* $B = (b_{ij})_{n\times n}$ *where* $b_{ij} = (b_{ijL}, b_{ijM}, b_{ijU})$ *be two triangular fuzzy number matrices of same order. Then the maximum operation on it is given by* $L_{max} = \max(A, B) = (\sup\{a_{ij}, b_{ij}\})$ *where* $\sup\{a_{ij}, b_{ij}\} = (\sup(a_{ijL}, b_{ijL}), \sup(a_{ijM}, b_{ijM}), \sup(a_{ijU}, b_{ijU}))$ *is the* ij^{th} *element of* $\max(A, B)$.

Definition 2.7 Arithmetic Mean (AM) for triangular fuzzy number
Let $A = (a_1, a_2, a_3)$ *be a triangular fuzzy number then* $AM(A) = \frac{a_1+a_2+a_3}{3}$. *The same condition holds for triangular fuzzy membership number.*

3 MEDICAL DIAGNOSIS UNDER FUZZY ENVIRONMENT

Let S be the set of symptoms of certain diseases, D is a set of diseases and P is a set of patients. The elements of triangular fuzzy number matrix are defined as

$$A = (a_{ij})_{m\times l} \text{ where } a_{ij} = (a_{ijL}, a_{ijM}, a_{ijU}) \text{ is the } ij^{th} \text{ element of } A, \quad 0 \leq a_{ijL} \leq a_{ijM} \leq a_{ijU} \leq 10. \tag{1}$$

Here a_{ijL} is the lower bound, a_{ijM} is the moderate value and a_{ijU} is the upper bound.

Procedure 3.1

Step 1: Construct a triangular fuzzy number matrix (F, D) over S, where F is a mapping given by $F : D \to \tilde{F}(S)$, $\tilde{F}(S)$ is a set of all triangular fuzzy sets of S. This matrix is denoted by R_0 which is the fuzzy occurrence matrix or symptom-disease triangular fuzzy number matrix.

Step 2: Construct another triangular fuzzy number matrix (F_1, S) over P, where F_1 is a mapping given by $F_1 : S \to \tilde{F}(P)$. This matrix is denoted by R_S which is the patient-symptom triangular fuzzy number matrix.

Step 3: Convert the elements of triangular fuzzy number matrix into its membership function as follows:

Membership function of $a_{ij} = (a_{ijL}, a_{ijM}, a_{ijU})$ is defined as

$$\mu_{a_{ij}} = \left(\frac{a_{ijL}}{10}, \frac{a_{ijM}}{10}, \frac{a_{ijU}}{10}\right), \quad \text{if } 0 \leq a_{ijL} \leq a_{ijM} \leq a_{ijU} \leq 10 \qquad (2)$$

where $0 \leq \frac{a_{ijL}}{10} \leq \frac{a_{ijM}}{10} \leq \frac{a_{ijU}}{10} \leq 1$.

Now the matrix R_0 and R_S are converted into triangular fuzzy membership matrices namely $(R_0)_{mem}$ and $(R_S)_{mem}$.

Step 4: Compute the following relation matrices.

$R_1 = (R_S)_{mem}(\cdot)(R_0)_{mem}$ it is calculated using Definition 2.5.

$R_2 = (R_S)_{mem}(\cdot)(J(-)(R_0)_{mem})$, where J is the triangular fuzzy membership matrix in which all entries are (1, 1, 1). $(J(-)(R_0)_{mem})$ is the complement of $(R_0)_{mem}$ and it is called as non symptom-disease triangular fuzzy membership matrix.

$R_3 = (J(-)(R_S)_{mem})(\cdot)(R_0)_{mem}$, where $(J(-)(R_S)_{mem})$ is the complement of R_S and it is called as non patient-symptom triangular fuzzy membership matrix.

R_2 and R_3 are calculated using subtraction operation and Definition 2.5.

$R_4 = \max\{R_2, R_3\}$. It is calculated using Definition 2.6.

The elements of R_1, R_2, R_3, R_4 is of the form $y_{ij} = (y_{ijL}, y_{ijM}, y_{ijU})$ where $0 \leq y_{ijL} \leq y_{ijM} \leq y_{ijU} \leq 1$.

$R_5 = R_1(-)R_4$. It is calculated using subtraction operation. The elements of R_5 is of the form $z_{ij} = (z_{ijL}, z_{ijM}, z_{ijU}) \in [-1, 1]$ where $z_{ijL} \leq z_{ijM} \leq z_{ijU}$.

Step 5: Calculate $R_6 = AM(z_{ij})$ and Row'_i = Maximum of i^{th} row which helps the decision maker to strongly confirm the disease for the patient.

Illustrative Example 3.1:

Suppose there are three patient's P_1, P_2 and P_3 in a hospital with symptoms temperature, headache, cough and stomach problem. Let the possible diseases relating to the above symptoms be viral fever and malaria.

Step 1: We consider the set $S = \{s_1, s_2, s_3, s_4\}$ as universal set where s_1, s_2, s_3 and s_4 represent the symptoms temperature, headache, cough and stomach problem respectively and the set $D = \{d_1, d_2\}$ where d_1 and d_2 represent the parameters viral fever and malaria respectively. Suppose that

$$\begin{aligned} F(d_1) = [&< e_1, (7, 8.5, 10) >, < e_2, (1, 2.5, 4) >, < e_3, (5, 5.5, 6) >, \\ &< e_4, (2, 3, 4) >] \\ F(d_2) = [&< e_1, (6, 7.5, 9) >, < e_2, (4, 5, 6) >, < e_3, (3, 4.5, 6) >, \\ &< e_4, (8, 9, 10) >] \end{aligned}$$

The triangular fuzzy number matrix (F, D) is a parameterized family $(F(d_1), F(d_2))$ of all triangular fuzzy number matrix over the set S and are determined from expert medical documentation. Thus the triangular fuzzy number matrix (F, D) represents a relation matrix R_0 and it gives an approximate description of the triangular fuzzy number matrix medical knowledge of the two diseases and their symptoms given by

$$R_0 = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \left[\begin{array}{cc} (7,8.5,10) & (6,7.5,9) \\ (1,2.5,4) & (4,5,6) \\ (5,5.5,6) & (3,4.5,6) \\ (2,3,4) & (8,9,10) \end{array} \right] \end{array}$$

Step 2: Again we take $P = \{p_1, p_2, p_3\}$ as the universal set where p_1, p_2 and p_3 represent patients respectively and $S = \{s_1, s_2, s_3, s_4\}$ as the set of parameters suppose that,
$F_1(s_1) = [< p_1, (6, 7.5, 9) >, < p_2, (3, 4, 5) >, < p_3, (6, 7, 8) >]$
$F_1(s_2) = [< p_1, (3, 4, 5) >, < p_2, (3, 5, 7) >, < p_3, (2, 4, 6) >]$
$F_1(s_3) = [< p_1, (8, 9, 10) >, < p_2, (2, 3, 4) >, < p_3, (5, 6, 7) >]$
$F_1(s_4) = [< p_1, (6, 7.5, 9) >, < p_2, (3, 4, 5) >, < p_3, (2, 3.5, 5) >]$
The triangular fuzzy number matrix (F_1, S) is another parameterized family of triangular fuzzy number matrix and gives a collection of approximate description of the patient-symptoms in the hospital. Thus the triangular fuzzy number matrix (F_1, S) represents a relation matrix R_S called patient-symptom matrix given by

$$R_S = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \left[\begin{array}{cccc} (6,7.5,9) & (3,4,5) & (8,9,10) & (6,7.5,9) \\ (3,4,5) & (3,5,7) & (2,3,4) & (3,4,5) \\ (6,7,8) & (2,4,6) & (5,6,7) & (2,3.5,5) \end{array} \right] \end{array}$$

Step 3:

$$(R_0)_{mem} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{cc} d_1 & d_2 \\ \left[\begin{array}{cc} (0.7,0.85,1) & (0.6,0.75,0.9) \\ (0.1,0.25,0.4) & (0.4,0.5,0.6) \\ (0.5,0.55,0.6) & (0.3,0.45,0.6) \\ (0.2,0.3,0.4) & (0.8,0.9,1) \end{array}\right] \end{array}$$

$$(R_S)_{mem} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \\ \left[\begin{array}{cccc} (0.6,0.75,0.9) & (0.3,0.4,0.5) & (0.8,0.9,1) & (0.6,0.75,0.9) \\ (0.3,0.4,0.5) & (0.3,0.5,0.7) & (0.2,0.3,0.4) & (0.3,0.4,0.5) \\ (0.6,0.7,0.8) & (0.2,0.4,0.6) & (0.5,0.6,0.7) & (0.2,0.35,0.5) \end{array}\right] \end{array}$$

Step 4: Computing the following relation matrices

$$R_1 = (R_S)_{mem}(\cdot)(R_0)_{mem} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{cc} d_1 & d_2 \\ \left[\begin{array}{cc} (0.6,0.75,0.9) & (0.6,0.75,0.9) \\ (0.3,0.4,0.5) & (0.3,0.5,0.6) \\ (0.6,0.7,0.8) & (0.6,0.7,0.8) \end{array}\right] \end{array}$$

$$R_2 = (R_S)_{mem}(\cdot)(J(-)(R_0)_{mem}) = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{cc} d_1 & d_2 \\ \left[\begin{array}{cc} (0.6,0.7,0.8) & (0.4,0.55,0.7) \\ (0.3,0.5,0.7) & (0.3,0.5,0.6) \\ (0.4,0.45,0.6) & (0.4,0.55,0.7) \end{array}\right] \end{array}$$

$$R_3 = (J(-)(R_S)_{mem})(\cdot)(R_0)_{mem} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{cc} d_1 & d_2 \\ \left[\begin{array}{cc} (0.1,0.25,0.4) & (0.4,0.5,0.6) \\ (0.5,0.6,0.7) & (0.5,0.6,0.7) \\ (0.3,0.4,0.5) & (0.5,0.65,0.8) \end{array}\right] \end{array}$$

$$R_4 = \max\{R_2, R_3\} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{cc} d_1 & d_2 \\ \left[\begin{array}{cc} (0.6,0.7,0.8) & (0.4,0.55,0.7) \\ (0.5,0.6,0.7) & (0.5,0.6,0.7) \\ (0.4,0.45,0.6) & (0.5,0.65,0.8) \end{array}\right] \end{array}$$

$$R_5 = R_1(-)R_4 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{cc} d_1 & d_2 \\ \left[\begin{array}{cc} (-0.2,0.05,0.3) & (-0.1,0.2,0.5) \\ (-0.4,-0.2,0) & (-0.4,-0.1,0.1) \\ (0,0.25,0.4) & (-0.2,0.05,0.3) \end{array}\right] \end{array}$$

Step 5:

$$R_6 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{cc} d_1 & d_2 \\ \left[\begin{array}{cc} 0.05 & 0.2 \\ -0.2 & -0.13 \\ 0.22 & 0.05 \end{array}\right] \end{array} \begin{array}{c} Row'_i = \text{Maximum of } i^{th} \text{ row} \\ 0.2 \\ -0.13 \\ 0.22 \end{array}$$

This can be represented in the form of a graph namely network as follows:

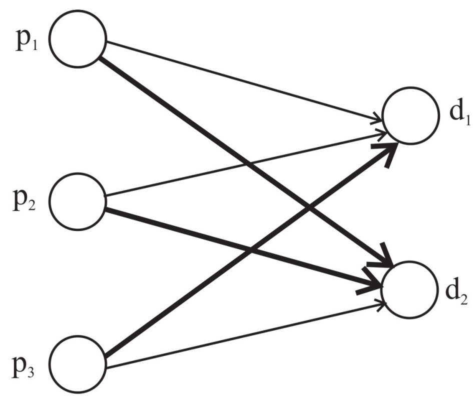

Figure 1: Fuzzy Medical Diagnosis Network

In the above network, nodes or vertices denote the patients and diseases, lengths or edges denote the assumption of diseases to the patients. The darken edges denotes the strong confirmation of disease to the patient.

4 DECISION MAKING UNDER FUZZY ENVIRONMENT

When we compare objects that are fuzzy or vague, we may have a situation where there is a contradiction of transitivity in the ranking. This form of non-transitive ranking can be accommodated by means of relativity function which is defined as a measurement of the membership value of choosing one variable over the other [6].

Definition 4.1 Relativity function
Let x and y be variables defined on a universal set X. The relativity function is denoted as $f(x/y)$ and is defined as

$$f(x/y) = \frac{\mu_y(x)(-)\mu_x(y)}{\max\{\mu_y(x), \mu_x(y)\}} \tag{3}$$

where $\mu_y(x)$ is the membership function of x with respect to y for triangular fuzzy number and $\mu_x(y)$ is the membership function of y with respect to x for triangular fuzzy number. Here $\mu_y(x)(-)\mu_x(y)$ is calculated using subtraction operation and $\max\{\mu_y(x), \mu_x(y)\}$ is calculated using Definition 2.6.

Definition 4.2 Comparison Matrix
Let $A = \{x_1, x_2, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n\}$ be a set of n variables defined on X. Form a matrix of relativity values $f(x_i/x_j)$ where x_i's for $i = 1$ to n, are n variables defined on an universe X. The matrix $C = (c_{ij})$ is a

square matrix of order n *is called the comparison matrix (or)* C*-matrix, with* $AM(f(x_i/x_j)) = \frac{AM(\mu_{x_j}(x_i)(-)\mu_{x_i}(x_j))}{AM(\max\{\mu_{x_j}(x_i),\mu_{x_i}(x_j)\})}$ *where* AM *denote the Arithmetic Mean and it is calculated using Definition 2.7. The comparison matrix is used to rank different fuzzy sets, the elements of* C*-matrix* $\in [-1, 1]$*. The smallest value in the* i^{th} *row of the comparison matrix, that is* $C'_i = \min\{f(x_i/X), i = 1 \text{ to } n\}$ *is the membership value of the* i^{th} *variable. The minimum of* $\{C'_i/i = 1 \text{ to } n\}$*, that is, the smallest value in each of the rows of the* C*-matrix will have the lowest weight for ranking purpose. Thus ranking the variables* $x_1, x_2, \ldots, x_n$ *are determined by ordering the membership values* $C'_1, C'_2, \ldots, C'_n$*.*

Procedure 4.1

Step 1: Gather the imprecise estimations needed for the problem which is in the form of triangular fuzzy number matrix using eqn. (1).

Step 2: Calculate the triangular membership matrix using eqn. (2).

Step 3: By using eqn. (3) let us calculate all of the relativity values $f(x_i/x_j)$. Form the comparison matrix using Definition 4.2, this gives the solution to the required problem. Here $f(x_i/x_j) = \frac{(0,0,0)}{(1,1,1)}$ for $i = j$.

Illustrative Example 4.1:

Step 1: Let us find out who resembles a father 'most' among his elder son (x_1), his younger son (x_2) and his daughter (x_3). We have the following imprecise estimations from family members.

$$A = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc} (10,10,10) & (7,8,9) & (3,5,7) \\ (3,5,7) & (10,10,10) & (6,7,8) \\ (3,5,7) & (2,4,6) & (10,10,10) \end{array} \right] \end{array}$$

This can be represented in the form of a graph (network) as follows:

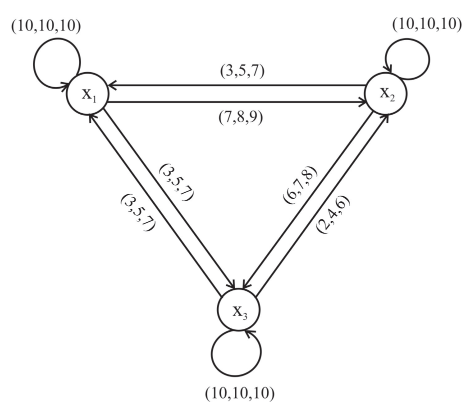

Figure 2: Fuzzy Decision Making Network

Step 2:

$$(A)_{mem} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc} (1,1,1) & (0.7,0.8,0.9) & (0.3,0.5,0.7) \\ (0.3,0.5,0.7) & (1,1,1) & (0.6,0.7,0.8) \\ (0.3,0.5,0.7) & (0.2,0.4,0.6) & (1,1,1) \end{array}\right] \end{array}$$

$\mu_{x_1}(x_1) = (1,1,1)$, $\mu_{x_2}(x_1) = (0.7,0.8,0.9)$, $\mu_{x_3}(x_1) = (0.3,0.5,0.7)$
$\mu_{x_1}(x_2) = (0.3,0.5,0.7)$, $\mu_{x_2}(x_2) = (1,1,1)$, $\mu_{x_3}(x_2) = (0.6,0.7,0.8)$
$\mu_{x_1}(x_3) = (0.3,0.5,0.7)$, $\mu_{x_2}(x_3) = (0.2,0.4,0.6)$, $\mu_{x_3}(x_3) = (1,1,1)$

Step 3:

$$f(x_1/x_1) = \frac{\mu_{x_1}(x_1)(-)\mu_{x_1}(x_1)}{\max\{\mu_{x_1}(x_1), \mu_{x_1}(x_1)\}} = \frac{(1,1,1)(-)(1,1,1)}{\max\{(1,1,1),(1,1,1)\}} = \frac{(0,0,0)}{(1,1,1)}$$

$$AM(f(x_1/x_1)) = \frac{0}{1} = 0$$

$$\begin{aligned} f(x_1/x_2) &= \frac{\mu_{x_2}(x_1)(-)\mu_{x_1}(x_2)}{\max\{\mu_{x_2}(x_1), \mu_{x_1}(x_2)\}} \\ &= \frac{(0.7,0.8,0.9)(-)(0.3,0.5,0.7)}{\max\{(0.7,0.8,0.9),(0.3,0.5,0.7)\}} \\ &= \frac{(0,0.3,0.6)}{(0.7,0.8,0.9)} \end{aligned}$$

$$AM(f(x_1/x_2)) = \frac{0.3}{0.8} = 0.375$$

$$\begin{aligned} f(x_1/x_3) &= \frac{\mu_{x_3}(x_1)(-)\mu_{x_1}(x_3)}{\max\{\mu_{x_3}(x_1), \mu_{x_1}(x_3)\}} \\ &= \frac{(0.3,0.5,0.7)(-)(0.3,0.5,0.7)}{\max\{(0.3,0.5,0.7),(0.3,0.5,0.7)\}} \\ &= \frac{(-0.4,0,0.4)}{(0.3,0.5,0.7)} \end{aligned}$$

$$AM(f(x_1/x_3)) = \frac{0}{0.5} = 0$$

$$f(x_2/x_1) = \frac{\mu_{x_1}(x_2)(-)\mu_{x_2}(x_1)}{\max\{\mu_{x_1}(x_2), \mu_{x_2}(x_1)\}} = \frac{(-0.4,-0.3,0)}{(0.7,0.8,0.9)}$$

$$AM(f(x_2/x_1)) = -0.29$$

$$f(x_2/x_2) = \frac{\mu_{x_2}(x_2)(-)\mu_{x_2}(x_2)}{\max\{\mu_{x_2}(x_2), \mu_{x_2}(x_2)\}} = \frac{(0,0,0)}{(1,1,1)}$$
$$AM(f(x_2/x_2)) = 0$$
$$f(x_2/x_3) = \frac{\mu_{x_3}(x_2)(-)\mu_{x_2}(x_3)}{\max\{\mu_{x_3}(x_2), \mu_{x_2}(x_3)\}} = \frac{(0,0.3,0.6)}{(0.6,0.7,0.8)}$$
$$AM(f(x_2/x_3)) = 0.43$$
$$f(x_3/x_1) = \frac{\mu_{x_1}(x_3)(-)\mu_{x_3}(x_1)}{\max\{\mu_{x_1}(x_3), \mu_{x_3}(x_1)\}} = \frac{(-0.4,0,0.4)}{(0.3,0.5,0.7)}$$
$$AM(f(x_3/x_1)) = 0$$
$$f(x_3/x_2) = \frac{\mu_{x_2}(x_3)(-)\mu_{x_3}(x_2)}{\max\{\mu_{x_2}(x_3), \mu_{x_3}(x_2)\}} = \frac{(-0.6,-0.3,0)}{(0.6,0.7,0.8)}$$
$$AM(f(x_3/x_2)) = -0.43$$
$$f(x_3/x_3) = \frac{\mu_{x_3}(x_3)(-)\mu_{x_3}(x_3)}{\max\{\mu_{x_3}(x_3), \mu_{x_3}(x_3)\}} = \frac{(0,0,0)}{(1,1,1)}$$
$$AM(f(x_3/x_3)) = 0$$

The comparison matrix $C = (c_{ij}) = AM(f(x_i/x_j))$ is given by

$$C = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc} 0 & 0.375 & 0 \\ -0.29 & 0 & 0.43 \\ 0 & -0.43 & 0 \end{array}\right] \end{array} \begin{array}{c} C_i' = \text{minimum of } i^{th} \text{ row} \\ 0 \\ -0.29 \\ -0.43 \end{array}$$

For this problem the ranking is x_1, x_2 and x_3. Hence the elder son resembles his father the 'most'.

5 CONCLUSION

Medicine is one of the field in which the applicability of fuzzy set theory was recognized quite early. The physician generally gathers knowledge about the patient from the past history, laboratory test result and other investigative procedures such as x-rays and ultra sonic rays etc. The knowledge provided by each of these sources carries with it varying degrees of uncertainty. Thus the best and most useful descriptions of disease entities often use linguistic terms that are vague.

As fuzzy decision making is a most important scientific, social and economic endeavour, there exist several major approaches within the theories of fuzzy decision making. Here we have used the ranking order to deal with the vagueness in imprecise determination of preferences.

Hence in this paper, Fuzzy set framework has been utilized in several different approaches to model the medical diagnostic process and decision making process.

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