STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/MC/VL64

B. Sc. DEGREE EXAMINATION, APRIL 2015
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : VECTOR SPACES AND LINEAR TRANSFORMATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS. **(10 X 2 =20)**

1. Define vector space.
2. Define homomorphism on vector spaces.
3. Prove that $L(S)$ is a subspace of V.
4. In $F^{(3)}$, prove that the vectors $(1,0,0), (0,1,0)$ and $(0,0,1)$ are linearly independent.
5. Define an inner product space.
6. If W is the subspace of V, then define the orthogonal complement of W.
7. Define an algebra.
8. If $T \in A(V)$ and V is finite dimensional over F, then define the rank of T
9. Define similarity transformation.
10. When a matrix A is said to be orthogonally diagonalizable.

SECTION –B

ANSWER ANY FIVE QUESTIONS. **(5x8 = 40)**

11. If V is a vector space over F, then prove that
    i) $\alpha 0 = 0$ for $\alpha \in F$ ii) $0v = 0$ for $v \in V$
    iii) $(-\alpha)v = -(\alpha v)$ for $\alpha \in F, v \in V$ iv) if $v \neq 0$, then $\alpha v = 0$ implies that $\alpha = 0$.

12. i) If F is a field of real numbers ,then show that the set of real-valued, continuous functions on the closed interval [0,1] forms a vector space over F.
    ii) Show also that those functions in part (i) for which all nth derivatives exists for n=1,2,3,…. form a subspace.

13. i) Define linearly independent vectors.
    ii) If $If\ v_1, v_{2,}, v_3, \dots \dots, v_n \in V$ are linearly independent, then prove that every element in their linear span has a unique representation in the form $\lambda_1 v_1 + \cdots \dots \dots \dots \dots + \lambda_n v_n$ with $\lambda_i \in F$.

14. State and prove Schwarz inequality.

15. Let V be the set of all continuous complex valued functions on the closed Interval [0,1].If $f(t), g(t) \in V$, define $(f(t), g(t)) = \int_0^1 f(t)\overline{g(t)}\, dt$, then show that this defines an inner product on V.

16. If V is finite dimensional over F, then prove that $T \in A(V)$ is invertible iff the constant term of the minimal polynomial for T is not zero.

17. Define matrix of a linear transformation of an n-dimensional vector space. Show that the matrix $A = \begin{pmatrix} 5 & -3 \\ 3 & -1 \end{pmatrix}$ is not diagonalizable.

SECTION –C

ANSWER ANY TWO QUESTIONS. (2x20 = 40)

18. i) If V is the internal direct sum of $U_1, U_2, \dots\dots\dots\dots., U_n$ then prove that V is Isomorphic to the external direct sum of $U_1, U_2, \dots\dots\dots\dots. U_n$.
 ii) If $v_1, \dots\dots\dots v_k$ are in V, then prove that either they are linearly independent or some v_k is a linear combination of the preceding ones, $v_1, \dots\dots\dots, v_{k-1}$.

19. i) If V is a finite dimentional inner product space , then prove that V has an orthonormal set as a basis.
 ii) Let F be the real field and let V be the set of polynomials in x over F of degree 2 or less. In V, define the inner product by $(p(x), q(x)) = \int_{-1}^{+1} p(x)q(x)dx.$ Then construct an orthonormal set corresponding to the basis $v_1 = 1, v_2 = x, v_3 = x^3$ of V.

20. i) If V is finite dimentional over F, then prove that $T \in A(V)$ is singular iff there exists a $v \neq 0$ in V such that $vT = 0$.
 ii) If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then prove that for any polynomial $q(x) \in F[X], q(\lambda)$ is a characteristic root of $q(T)$.
 iii) Let A be square matrix. Then prove that A is orthogonally diagonalizable iff it is a symmetric matrix.

.

⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄