STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/MC/SF44

B. Sc. DEGREE EXAMINATION, APRIL 2015
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : MAJOR CORE
PAPER : SEQUENCES AND SERIES, FOURIER SERIES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(10×2=20)**

1. If $f(x)=1+x^3, 1\le x\le 3,\ g(x)=1+x^3, 1\le x\le 4,$ out of f and g which one is the extension of the other? Why?
2. Let A be a subset of S. Define the characteristic function of A.
3. Find the limit of the sequence $\{s_n\}_{n=1}^{\infty}$ where $s_n=\frac{1}{n}$.
4. Give two examples of a bounded sequence.
5. Define limit superior of a sequence.
6. Define Cauchy sequence.
7. Define alternating series and give an example.
8. Prove that $\sum_{n=1}^{\infty}\frac{1-n}{1+2n}$ is divergent.
9. Define odd function and give an example.
10. Find the Fourier coefficient a_0 for $f(x)=x(2\pi-x)$ in the interval $(0,2\pi)$.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5×8=40)**

11. If $f: A\to B$ and if $X\subset A, Y\subset A,$ Prove that $f(X\cup Y)=f(X)\cup f(Y)$.
12. Using the method of first principle find the limit of $\left\{\frac{2n}{n+4n^{\frac{1}{2}}}\right\}_{n=1}^{\infty}$
13. Prove that a non decreasing sequence which is bounded above is convergent.
14. If $\sum_{n=1}^{\infty}a_n$ is a convergent series, Prove that $\lim_{n\to\infty}a_n=0$.
15. Obtain the half range cosine series for $f(x)=x^2$ in $(0,\pi)$.
16. If the sequence $\{s_n\}_{n=1}^{\infty}$ of real numbers is convergent, Prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.
17. Test the convergence of the series $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots\infty$.

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2×20=40)**

18. a) Prove that the set of rational numbers in $[0,1]$ is uncountable.

b) Prove that the sequence $\{s_n\}_{n=1}^{\infty}$ where $s_n = \dfrac{n^2+1}{2n^2+5}, \forall n \in N$ converges to $\dfrac{1}{2}$.

(10+10)

19. a) If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, $\lim\limits_{n\to\infty} s_n = L,$ and $\lim\limits_{n\to\infty} t_n = M$,

Prove that $\lim\limits_{n\to\infty} s_n t_n = LM$.

b) State and prove ratio test.

(10+10)

20. a) Find the half range sine series for $f(x) = \begin{cases} -x, -\pi \le x \le 0, \\ x, 0 \le x \le \pi \end{cases}$.

b) Determine the Fourier series expansion for $f(x) = \dfrac{\pi}{2} - x$ in the interval $(0, 2\pi)$.

(10+10)

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