STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/MC/IC24

B. Sc. DEGREE EXAMINATION, APRIL 2015
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : MAJOR CORE
PAPER : INTEGRAL CALCULUS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(10X2=20)**

1. Evaluate $\int \frac{x^2}{x+2}\, dx$.
2. Prove $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$.
3. Integrate $\frac{1}{\sqrt{25-x^2}}$ with respect to x.
4. Integrate xe^x with respect to x.
5. Evaluate $\int_1^2 \int_2^5 xy\, dydx$.
6. Suppose R is the region bounded by $y = f_1(x), y = f_2(x)$ and the coordinated $x = a$, $x = b$ then how to find $\iint_R f(x,y)dA$
7. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^a dr\, d\theta$.
8. If $x = r\cos\theta; y = r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$.
9. Prove $\Gamma(n+1) = n\Gamma(n)$.
10. Define Beta function.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5X8=40)**

11. Evaluate $\int \frac{2x+3}{x^2+x+1}\, dx$.
12. Integrate $x^3\cos 2x$ with respect to x.
13. Prove that $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} = \frac{\pi}{4}$.

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14. Find the value of $\iint xy\, dy\, dx$ taken over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

15. Evaluate the following integral by change of order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy\, dx$.

16. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

17. Evaluate $\int_0^\infty \frac{x}{1+x^6}\, dx$.

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2X20=40)**

18. a) Evaluate $\int \frac{1}{x^2 - a^2}\, dx$.

    b) Find the value of $\int \frac{2\, dx}{(1-x)(1+x^2)}$

    c) Evaluate $\int \frac{6x+5}{\sqrt{6+x-2x^2}}\, dx$. (6 + 7 + 7)

19. a) Prove that $\int_0^{\frac{\pi}{4}} \log(1 + \tan\theta)\, d\theta = \frac{\pi}{8}\log 2$.

    b) Change the order of integration and hence evaluate

    $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\, dydx$. (10 + 10)

20. a) Evaluate $\int_0^\pi \int_0^{a(1+\cos\theta)} r\, dr\, d\theta$

    b) Prove that $\beta(m,n) = \beta(n,m)$

    c) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (6+4+10)