STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011–12)

SUBJECT CODE : 11MT/MC/GC64

B. Sc. DEGREE EXAMINATION, APRIL 2015
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : GRAPH THEORY AND COMBINETORICS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A

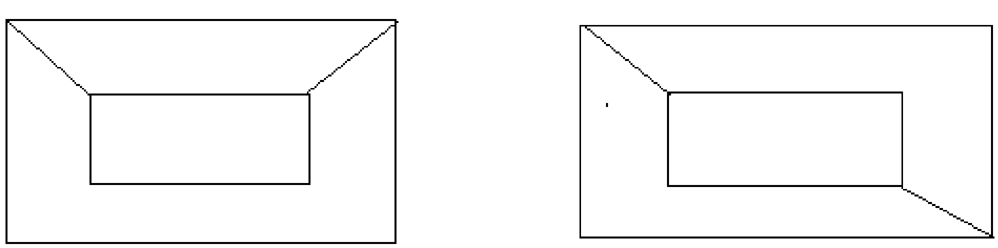
ANSWER ALL QUESTIONS: **10 X 2 = 20**

1. Find the number of vertices and edges of $K_{m,n}$.
2. Prove that every cubic graphs has an even number of points.
3. Define centre of a tree.
4. Give a graph which is Hamiltonian but not Eulerian.
5. Write the chromatic number of K_n .
6. If $D = (\{1,2,3,4\}, \{(1,2), (2,3), (1,3), (3,1)\})$ is a digraph, find the indegree and outdegree of each of the vertices.
7. How many minimum number of students required to ensure that a class includes at least two students whose names begin with the same letter of the English alphabet?
8. State inclusion-exclusion principle.
9. For the sequence $X = \{x_1, x_2, \ldots, x_{2n}\}$ find the number of derangements of X such that the first n elements of each derangement are the first n elements of X.
10. Obtain the ordinary generating function of
    (i) <1,-1,1,-1,…> (ii) <0,1,2,3,…>

SECTION-B

ANSWER ANY FIVE QUESTIONS: **5 X 8 = 40**

11. When are graphs said to be isomorphic. Prove that the following graphs are not isomorphic.



12. Prove that any self complimentary graph has $4n$ or $4n+1$ points.
13. Show that every tree has a centre containing one point or two adjacent points..
14. State and prove Euler's theorem for planar graphs.
15. Prove that $K_{3,3}$ is not planar.

16. State and prove Sieve formula.

17. Find the sequences corresponding to the ordinary generating functions

(a) $(3+x)^3$ (b) $3x^3+e^{2x}$ (c) $2x^2(1-x)^{-1}$

SECTION-C

ANSWER ANY TWO QUESTIONS: 2 X20 = 40

18. (i) Let G be a (p,q) graph. Prove that the following statements are equivalent.

(a) G is a tree

(b) Any two point of G is joined by a unique poth.

(c) G is connected and $p=q+1$

(d) G is acyclic and $p=q+1$.

(ii) State and prove five color theorem. (10 + 10)

19. a) Define adjacency matrix of a graph. Cite an example to explain the same. Also prove that given a symmetric binary matrix A, you can construct a graph having A as its adjacency matrix.

b) Write the combinatorial proof for

(i) Pascal's identity

(ii) Newton's identity

(iii) $C(2n,2)=2C(n,2)+n^2$

20. (i) Find the coefficients ofx^{27} in $(a)(x^4+x^5+x^6+\cdots)^5$

$(b)(x^4+2x^5+\quad 3x^6+\cdots)^5$

(ii) Define Fibinacci sequence and write the recurrence relation and hence show that for $n=0,1,2,\dots..f(n)=\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1}-\left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]$. (8 + 12)