**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**
**(For candidates admitted from the academic year 2011–12 & thereafter)**

**SUBJECT CODE : 11MT/MC/CA64**

**B. Sc. DEGREE EXAMINATION, APRIL 2015**
**BRANCH I – MATHEMATICS**
**SIXTH SEMESTER**

**COURSE : MAJOR CORE**
**PAPER : COMPLEX ANALYSIS**
**TIME : 3 HOURS** **MAX. MARKS : 100**

**SECTION-A**

**ANSWER ALL QUESTIONS:** **10 X 2 = 20**

1. Find the real and imaginary parts of $f(z) = 2\bar{z}^2 + 1$.
2. Is $f(z) = \bar{z}$ differentiable? Justify your answer.
3. Find the centre and radius of the image circle of $|z-3| = 5$ under $w = \dfrac{1}{z}$.
4. Define cross ratio of four points $z_1, z_2, z_3, z_4$ in the extended complex plane.
5. State maximum modulus theorem.
6. Evaluate: $\int_C \dfrac{\sin z}{z-1} dz$ where $C$ is the positively oriented circle $|z| = 2$.
7. State Laurent's theorem.
8. Locate and classify the singularity of $f(z) = \dfrac{1}{z}$.
9. Find the residue of $f(z) = e^{1/z}$ at its isolated essential singularity $z = 0$.
10. State Argument theorem.

**SECTION-B**

**ANSWER ANY FIVE QUESTIONS:** **5 X 8 = 40**

11. Prove that an analytic function with constant modulus must reduce to a constant.
12. Find the bilinear transformation which maps the points $z = -1, 1, \infty$ respectively on $w = -i, -1, i$.
13. Discuss the transformation $w = e^z$.
14. Evaluate $\int_C \dfrac{e^z}{(z+2)(z+4)^2} dz$ where $C$ is the positively oriented circle $|z| = 3$.
15. State and prove Cauchy's integral formula.
16. Find the Laurent series expansion of $f(z) = \dfrac{-1}{(z-1)(z-2)}$ in the region $1 < |z| < 2$.
17. Evaluate: $\int_0^{2\pi} \dfrac{d\theta}{2 + \cos\theta}$.

**..2**

## SECTION-C

**ANSWER ANY TWO QUESTIONS: 2 X20 = 40**

18. a) Prove that $f(z) = e^{\bar{z}}$ is nowhere differentiable.
    b) Derive C-R equations in polar coordinates.
    c) Find the image of the circle $|z-3i| = 3$ under the map $w = \frac{1}{z}$. $(6+10+4)$

19. a) State and prove fundamental theorem of algebra.
    b) State and prove Taylor's theorem. $(10+10)$

20. a) State and prove Rouche's theorem.
    b) Find the residue of $f(z) = \frac{z+1}{z^2 - 5z + 6}$ at its poles.
    c) Using the method of contour integration evaluate $\int_0^\infty \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx.$
    $(7+5+8)$